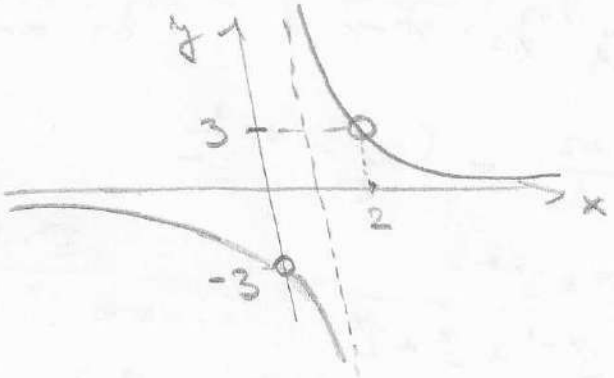
1. $f(x) = \left(\frac{6}{x-2} + \frac{3}{1-x}\right) : \left(1 + \frac{2}{x-2}\right) =$

$$= \frac{6(1-x)+3(x-2)}{(x-2)(1-x)} : \frac{x-2+2}{x-2} = \frac{6-6x+3x-6}{\cancel{(x-2)}(1-x)} \cdot \frac{\cancel{x-2}}{x} =$$

$$= \frac{(-3\cancel{x})}{1-x} \cdot \frac{1}{\cancel{x}} = -\frac{3}{1-x} = \frac{3}{x-1}, \quad x \neq 1, 2, 0$$

$D_f = \mathbb{R} \setminus \{0; 1; 2\}$

$H_f = \mathbb{R} \setminus \{-3; 0; 3\}$



2. $g_1(x) = \sqrt{\frac{3x-6}{7-x}}$

$\frac{3x-6}{7-x} \geq 0$

$3x-6$	$-$	$+$	$+$
$7-x$	$+$	$+$	$-$
	$-$	$+$	$-$
$-\infty$	2	7	$+\infty$

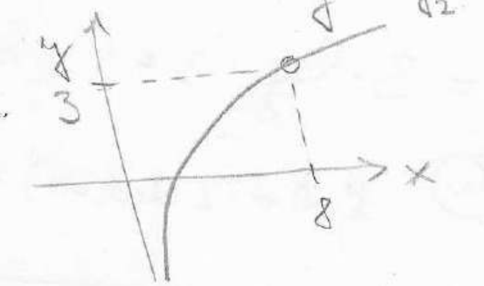
$D_{g_1} = \langle 2, 7)$

$g_2(x) = \log_{\frac{1}{2}}(3 - \log_2 x)$

I. $x > 0 \quad \wedge \quad$ II. $3 - \log_2 x > 0$

$\log_2 x < 3$

$x < 8$



$D_{g_2} = (0, 8)$

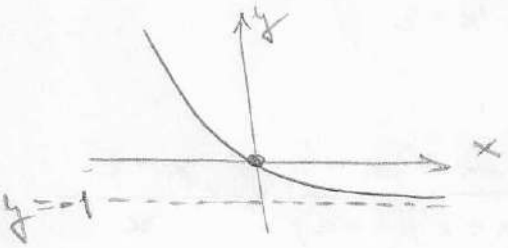
3. $\forall x \in \mathbb{R}: h_1(-x) = 3^{-x} + 3^{-(-x)} = 3^x + 3^{-x} = h_1(x)$

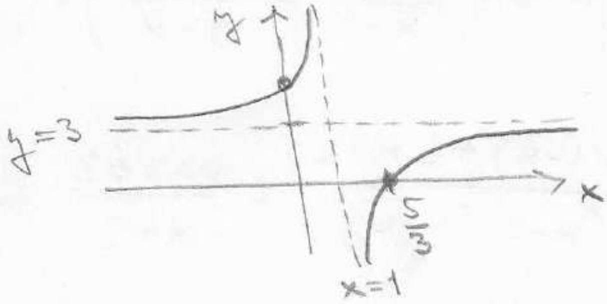
$\Rightarrow h_1$ je sudá

$\forall x \in \mathbb{R}: h_2(x) = \sin(2 \cdot (-x)) = \sin(-(2x)) = -\sin 2x = -h_2(x)$

$\Rightarrow h_2$ je lichá

4. $f_1(x) = 2^{-x} - 1$ $\qquad$ $f_2(x) = 3 + \frac{2}{1-x}$





$D_{f_1} = \mathbb{R}$, $H_{f_1} = (-1, \infty)$

$D_{f_2} = \mathbb{R} \setminus \{1\}$, $H_{f_3} = \mathbb{R} \setminus \{3\}$

5. a) $\lim\limits_{x \to 1} \log_9 \left(\frac{x^2+x-2}{x^2-x} \right) = \log_9 \left(\lim\limits_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{x\cancel{(x-1)}} \right) =$

$= \log_9 \frac{1+2}{1} = \log_9 3 = \underline{\underline{\frac{1}{2}}}$

b) $\lim\limits_{x \to +\infty} \frac{2^x + 3^{x+2}}{3^{x-1} + 3^x + 2^x} = \lim\limits_{x \to +\infty} \frac{\cancel{3^x} \cdot \left(\left(\frac{2}{3}\right)^x + 3^2 \right)}{\cancel{3^x} \cdot \left(3^{-1} + 1 + \left(\frac{2}{3}\right)^x \right)} = \frac{0+9}{\frac{1}{3}+1+0} = \underline{\underline{\frac{27}{4}}}$

c) $\lim\limits_{x \to +\infty} \frac{6x+1}{2x+7} \cdot \log_{\frac{1}{2}} \left(\frac{3x^2}{x^2+1} + 2^{\frac{1}{x}} \right) = \lim\limits_{x \to \infty} \frac{6x+1}{2x+7} \cdot \log_{\frac{1}{2}} \left(\lim\limits_{x \to \infty} (-\text{||}-) \right) =$

$= \underbrace{\lim\limits_{x \to +\infty} \frac{\cancel{x}\left(6 + \frac{1}{x}\right)}{\cancel{x}\left(2 + \frac{7}{x}\right)}}_{\frac{6+0}{2+0} = 3} \cdot \log_{\frac{1}{2}} \left(\underbrace{\lim\limits_{x \to +\infty} \frac{3\cancel{x^2}}{\cancel{x^2}\left(1 + \frac{1}{x^2}\right)}}_{\frac{3}{1+0} = 3} + \underbrace{\lim\limits_{x \to \infty} 2^{\frac{1}{x}}}_{2^0 = 1} \right) =$

$= 3 \cdot \log_{\frac{1}{2}} (3+1) = 3 \cdot \log_{\frac{1}{2}} 4 = 3 \cdot (-2) = \underline{\underline{-6}}$

6. $f'(x) = \left(2\sin(3x) + 3x^4 \right)' = \underline{6\cos(3x) + 12x^3}$

$g'(x) = \Big(\underbrace{\sqrt[3]{x \sqrt{x \sqrt{x}}}}_{} \Big)' = \left(x^{\frac{7}{12}} \right)' = \underline{\frac{7}{12} \cdot x^{-\frac{5}{12}}}$

$\left(x \cdot \left(x \cdot x^{\frac{1}{2}} \right)^{\frac{1}{2}} \right)^{\frac{1}{3}} = \Big(\underbrace{x \cdot \left(x^{\frac{3}{2}} \right)^{\frac{1}{2}}}_{x^{\frac{3}{4}}} \Big)^{\frac{1}{3}} = \left(x^{\frac{7}{4}} \right)^{\frac{1}{3}} = x^{\frac{7}{12}}$

$h'(x) = \left(\ln\left(\frac{3x+2}{3x-2} \right) \right)' = \frac{1}{\frac{3x+2}{3x-2}} \cdot \frac{3(3x-2) - (3x+2) \cdot 3}{(3x-2)^2} = \frac{-12}{(3x+2)(3x-2)}$

$= -\frac{12}{9x^2-4}$