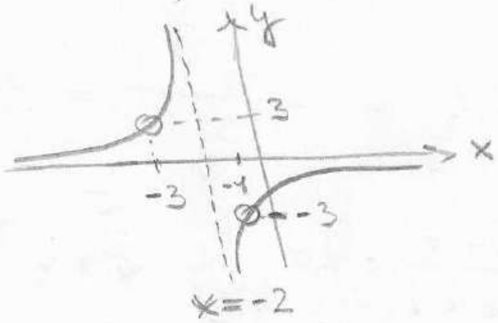
(1.) $f(x) = \left(\frac{3}{x+2} - \frac{6}{x+3}\right) : \left(1 - \frac{2}{x+3}\right) =$

$= \frac{3(x+3) - 6(x+2)}{(x+2)(x+3)} : \frac{x+3-2}{x+3} = \frac{3x+9-6x-12}{(x+2)(x+3)} : \frac{x+1}{x+3} =$

$= \frac{-3x-3}{(x+2)\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{x+1} = \frac{(-3)\cancel{(x+1)}}{(x+2)\cancel{(x+1)}} = -\frac{3}{x+2}$, $x \neq -2, -3, -1$

$D_f = \mathbb{R} \setminus \{-3; -2; -1\}$

$H_f = \mathbb{R} \setminus \{3; 0; -3\}$



(2.) $g_1(x) = \sqrt{\frac{2x-8}{9-x}}$

$\frac{2x-8}{9-x} \geq 0$

$D_{g_1} = \langle 4, 9)$

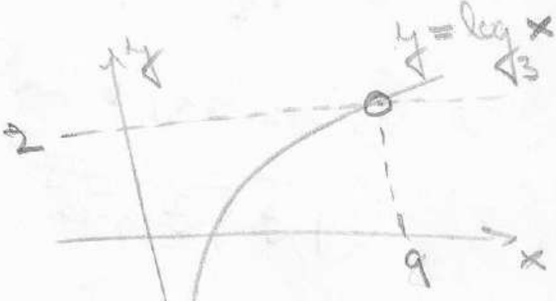
	$-\infty$	4		9	$+\infty$
$2x-8$	$-$	•	$+$		$+$
$9-x$	$+$		$+$	∘	$-$
$\frac{2x-8}{9-x}$	$-$		$+$		$-$

$g_2(x) = \log_{\frac{1}{3}}(2 - \log_3 x)$

$x > 0 \quad \wedge \quad 2 - \log_3 x > 0$

$\log_3 x < 2$

$x < 9$



$D_{g_2} = (0, 9)$

(3.) $h_1(-x) = 2^{-x} + 2^{-(-x)} = 2^x + 2^{-x} = h_1(x) \Rightarrow h_1$ je <u>sudá</u>

$\forall x \in \mathbb{R}$: $h_2(-x) = \sin(3 \cdot (-x)) = \sin(-(3x)) = -\sin(3x) = -h_2(x)$

$\Rightarrow h_2$ je <u>lichá</u>

4. $f_1(x) = \log_{\frac{1}{2}}(1-x)$ $\qquad D_{f_1} = (-\infty, 1)$

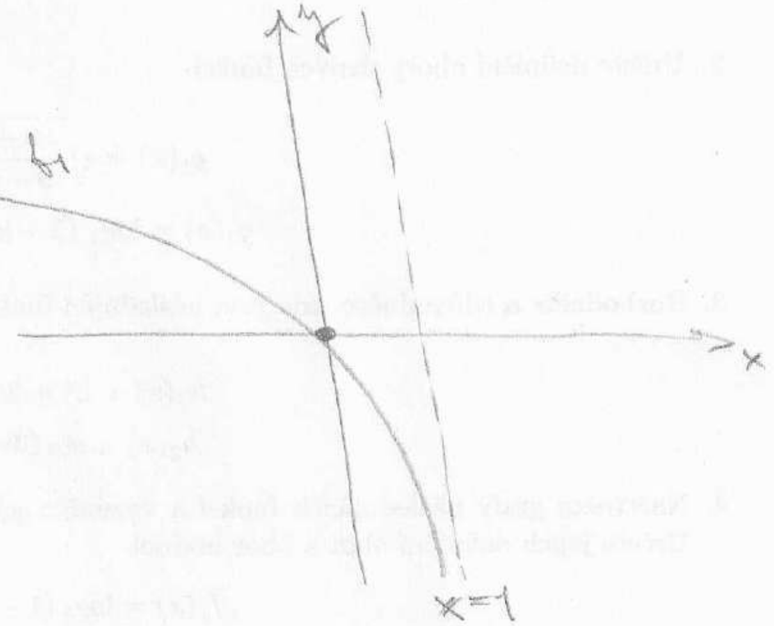
$H_{f_1} = \mathbb{R}$

$1-x > 0$

$x < 1$

$f_1(0) = \log_{\frac{1}{2}} 1 = 0$

$P_x = P_y = [0,0]$



$f_2(x) = 2 + \frac{4}{1-x}$ $\qquad D_{f_2} = \mathbb{R} \setminus \{1\}$

$H_{f_2} = \mathbb{R} \setminus \{2\}$

$f_2(0) = 2 + \frac{4}{1-0} = 6$

$f_2(x) = 0 \iff 2 + \frac{4}{1-x} = 0$

$\frac{4}{1-x} = -2$

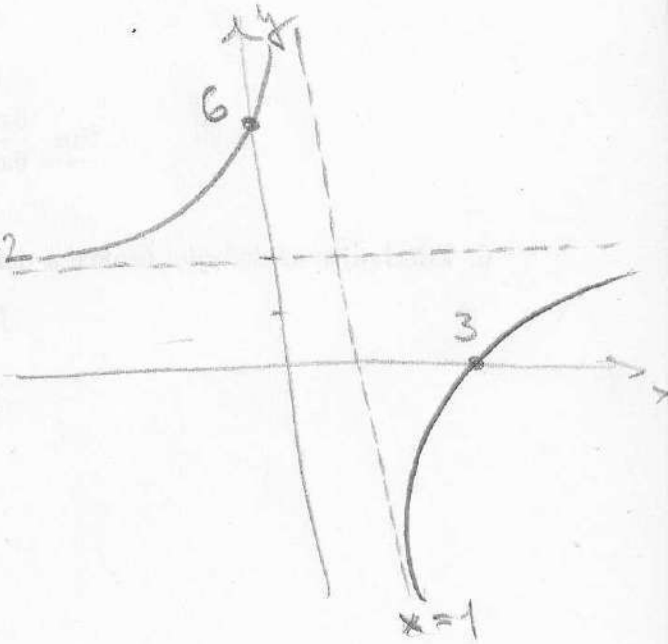
$4 = (-2)(1-x)$

$4 = -2 + 2x$

$6 = 2x$

$x = 3$

$P_x = [3;0]$, $P_y = [0;6]$



(5a) $\lim\limits_{x\to 1} \log_{16}\left(\frac{x^2+2x-3}{x^2-x}\right) = \log_{16}\left(\lim\limits_{x\to 1}\frac{(x+3)\cancel{(x-1)}}{x\cancel{(x-1)}}\right) =$

$= \log_{16}\frac{1+3}{1} = \log_{16} 4 = \frac{1}{2}$

(5b) $\lim\limits_{x\to+\infty}\frac{2^x+3^x+3^{x+1}}{2^x+3^x+3^{x-1}} = \lim\limits_{x\to\infty}\frac{\cancel{3^x}\cdot\left(\left(\frac{2}{3}\right)^x+1+3\right)}{\cancel{3^x}\cdot\left(\left(\frac{2}{3}\right)^x+1+\frac{1}{3}\right)} =$

$= \frac{0+1+3}{0+1+\frac{1}{3}} = \frac{4}{\frac{4}{3}} = \frac{\cancel{4}}{1}\cdot\frac{3}{\cancel{4}} = 3$

(5c) $\lim\limits_{x\to\infty}\frac{3x+7}{6x+1}\cdot\log_{\frac{1}{2}}\left(\frac{6x^2}{x^2+1}+9^{\frac{1}{x}}+1\right) =$

$= \lim\limits_{x\to+\infty}\frac{\cancel{x}\cdot(3+\frac{7}{x})}{\cancel{x}\cdot(6+\frac{1}{x})}\cdot\lim\limits_{x\to+\infty}\log_{\frac{1}{2}}(-\shortparallel-)$

$= \frac{3+0}{6+0}\cdot\log_{\frac{1}{2}}\Big(\underbrace{\lim\limits_{x\to+\infty}\left(\frac{\cancel{x^2}\cdot 6}{\cancel{x^2}\cdot(1+\frac{1}{x^2})}\right)}_{\frac{6}{1+0}=6} + \underbrace{\lim\limits_{x\to\infty} 9^{\frac{1}{x}}}_{=9^0=1} + 1\Big)$

$= \frac{1}{2}\log_{\frac{1}{2}}(6+1+1) = \frac{1}{2}\cdot\log_{\frac{1}{2}} 8 = -\frac{3}{2}$

(6a) $(3\cos(4x)+2x^5)' = -12\sin(4x)+10x^4$

(6b) $\left(\sqrt{x^3\sqrt{x^3\sqrt{x^3}}}\right)' = \left(x^{\frac{21}{8}}\right)' = \frac{21}{8}\cdot x^{\frac{13}{8}}$

$= \Big(x^3\cdot\big(\underbrace{x^3\cdot(x^3)^{\frac{1}{2}}}_{x^{3+\frac{3}{2}}}\big)^{\frac{1}{2}}\Big)^{\frac{1}{2}} = \left(x^3\cdot x^{\frac{9}{4}}\right)^{\frac{1}{2}} = \underbrace{\left(x^{\frac{21}{4}}\right)^{\frac{1}{2}}}_{x^{\frac{21}{8}}}$

(6c) $\left(\ln\left(\frac{2x+1}{2x-1}\right)\right)' = \left(\frac{2x+1}{2x-1}\right)'\cdot\frac{1}{\frac{2x+1}{2x-1}} =$

$= \frac{2\cdot(2x-1)-2(2x+1)}{(2x-1)^{\cancel{2}}}\cdot\frac{\cancel{2x-1}}{2x+1} = \frac{4x-2-4x-2}{(2x-1)(2x+1)} = -\frac{4}{4x^2-1}$