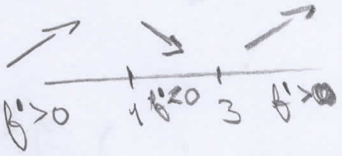
(A)

1) $f(x) = x^3 - 6x^2 + 9x$

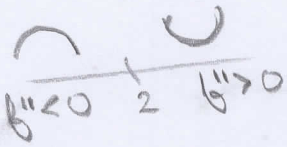
$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$



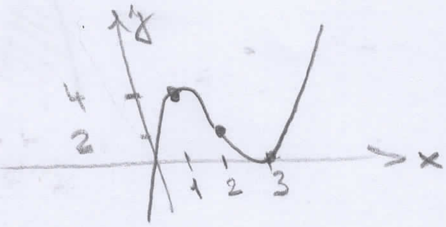
$f''(x) = 6x - 12 = 6(x-2)$

$f(1) = 4$ lok. max.

$f(3) = 0$ lok. min.

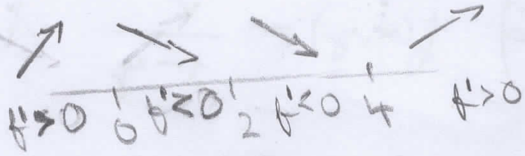


$f(2) = 2$



2) $g(x) = \dfrac{x^2}{x-2}$ $\quad D = \mathbb{R} - \{2\}$

$g'(x) = \dfrac{2x(x-2) - x^2}{(x-2)^2} = \dfrac{x^2 - 4x}{(x-2)^2} = \dfrac{x(x-4)}{(x-2)^2}$



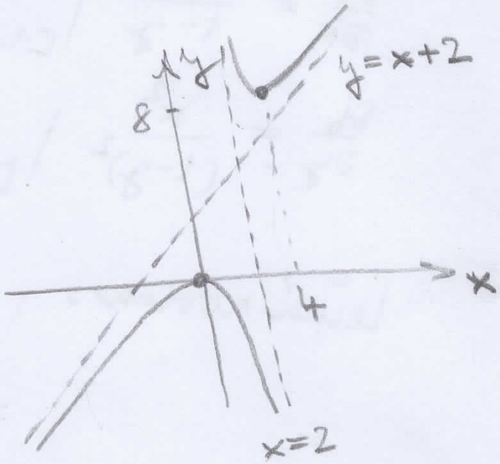
$g(0) = 0$ lok. max.

$g(4) = 8$ lok. min.

$k = \lim\limits_{x \to \pm\infty} \dfrac{g(x)}{x} = 1$

$q = \lim\limits_{x \to \pm\infty} (g(x) - k \cdot x) = \lim \left(\dfrac{x^2}{x-2} - x \right)$

$= \lim\limits_{x \to \pm\infty} \dfrac{\cancel{x^2} - \cancel{x^2} + 2x}{x-2} = 2$



3) $\lim\limits_{x \to 0} \dfrac{x \cdot e^x - x}{1 - x - e^{-x}} \overset{\text{L.P.}}{=} \lim\limits_{x \to 0} \dfrac{e^x + xe^x - 1}{-1 + e^{-x}} \overset{\text{L.P.}}{=} \lim\limits_{x \to 0} \dfrac{e^x + e^x + xe^x}{-e^{-x}} = \underline{\underline{-2}}$

4) $f(x) = \ln(1+6x) \quad \Big|_{x=0} = 0$

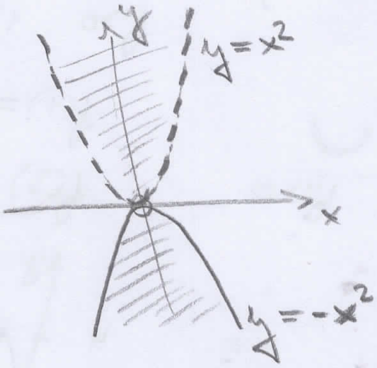
$f'(x) = \dfrac{6}{1+6x} \quad \Big|_{x=0} = 6$

$f''(x) = -\dfrac{36}{(1+6x)^2} \quad \Big|_{x=0} = -36$

$\Bigg\}\ T(x) = 6x - \dfrac{36}{2!}x^2 = \underline{\underline{6x - 18x^2}}$

5) $f(x,y) = \sqrt{\dfrac{y+x^2}{y-x^2}}$ $\qquad \dfrac{y+x^2}{y-x^2} \geq 0 \quad \Longleftrightarrow \quad y \geq -x^2 \wedge y \geq x^2$

$\vee \left(y \leq -x^2 \wedge y \leq x^2 \right)$



6) $f(x,y) = \dfrac{x^2}{1-y} \Big|_{[1,0]} = 1$

$\dfrac{\partial f}{\partial x} = \dfrac{2x}{1-y} \Big|_{[1,0]} = 2$

$\dfrac{\partial f}{\partial y} = \dfrac{x^2}{(1-y)^2} \Big|_{[1,0]} = 1$

$\Big\} \ \nabla f(1,0) = (2,1)$

Tečná rovina: $\boxed{z-1 = 2(x-1) + 1\cdot(y-0)}$

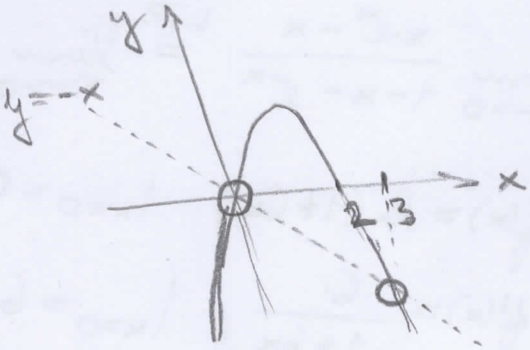
$z = 2x + y - 1$

7) $f(x,y) = \dfrac{x^2+3y}{x+y} = 2$

$x^2 + 3y = 2x + 2y$

$y = -x^2 + 2x$

$D_f = \{(x,y) \in \mathbb{R}^2 ;\ y \neq -x\}$



$-x \neq -x^2 + 2x$

$x^2 - 3x \neq 0$

$x(x-3) \neq 0 \quad \Rightarrow \quad x \neq 0 \wedge x \neq 3$