1) $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & 3 & -2 \end{pmatrix}$

SARRUSOVO PRAVIDLO

$$\det A = \begin{vmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & 3 & -2 \end{vmatrix} \begin{matrix} 1 & 2 & -2 \\ 2 & 1 & 2 \end{matrix} = 1\cdot 1\cdot(-2) + 2\cdot 3\cdot(-2) + 2\cdot 2\cdot 2 - \left(2\cdot 1\cdot(-2) + 1\cdot 3\cdot 2 + 2\cdot 2\cdot(-2)\right) =$$

$$= -2 - 12 + 8 - (-4 + 6 - 8) =$$

$$= -6 - (-6) = -6 + 6 = \underline{\underline{0}}$$

$\Rightarrow$ matice je singulární

$\Rightarrow$ inverze A^{-1} neexistuje

Hodnost: $\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & 3 & -2 \end{pmatrix} \begin{matrix} |\cdot(-2) \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 2 & -2 \\ 0 & -3 & 6 \\ 0 & -1 & 2 \end{pmatrix} |\cdot\left(-\frac{1}{3}\right) \sim \begin{pmatrix} 1 & 2 & -2 \\ 0 & -3 & 6 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$

$\Rightarrow h(A) = \underline{\underline{2}}$

Existují čísla $a, b \in \mathbb{R}$ takové, že:

$$\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = a\cdot\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + b\cdot\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad ?$$

neboli
$$\begin{aligned} a + 2b &= 2 \\ 2a + b &= 3 \\ -2a + 2b &= -2 \end{aligned}$$

$3b = 1 \Rightarrow \boxed{b = \frac{1}{3}}$ $\qquad 2a + \frac{1}{3} = 3 \quad |-\frac{1}{3}$

$2a = \frac{8}{3} \quad |:2$

$\boxed{a = \frac{4}{3}}$

dosazením do 1. rovnice ověříme, zda je splněna: $a + 2b = \frac{4}{3} + 2\cdot\frac{1}{3} = \frac{6}{3} = 2$ ✓

Ano, $\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \frac{4}{3}\cdot\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \frac{1}{3}\cdot\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

2) Řekneme, že λ je vlastním číslem matice B pokud existuje nenulový vektor $\vec{x}$ (vlastní vektor) tak, že $\boxed{B\vec{x} = \lambda\vec{x}}$ neboli $(B-\lambda I)\vec{x} = \vec{0}$

$\iff \boxed{\det(B-\lambda I) = 0}$

$$B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$$

$$\det(B-\lambda I) = \begin{vmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 1\cdot 2$$

$$= 2 - 2\lambda - \lambda + \lambda^2 - 2$$

$$= \lambda^2 - 3\lambda = \lambda(\lambda-3) = 0$$

$$\iff \underline{\underline{\lambda = 0}} \vee \underline{\underline{\lambda = 3}}$$

$\underline{\lambda = 0}$: $(B - 0\cdot I)\vec{x} = \vec{0}$

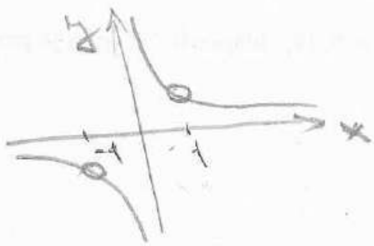
$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 2 & 0 \end{array}\right) \quad \vec{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\underline{\lambda = 3}$: $(B - 3I)\vec{x} = \vec{0}$

$$\left(\begin{array}{cc|c} -2 & 1 & 0 \\ 2 & -1 & 0 \end{array}\right) \quad \vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

3) $f(x)=\left(\frac{2}{x-1}-\frac{1}{x}\right):\left(1+\frac{2}{x-1}\right)=\frac{2x-(x-1)}{x(x-1)}:\frac{x-1+2}{x-1}$

$=\frac{x+1}{x(x-1)}:\frac{x+1}{x-1}=\frac{\cancel{x+1}}{x\cancel{(x-1)}}\cdot\frac{\cancel{x-1}}{\cancel{x+1}}=\frac{1}{x}$, $x\neq 0$, $x\neq\pm 1$



$D_f=\mathbb{R}-\{\pm 1,0\}$

$H_f=\mathbb{R}-\{\pm 1,0\}$

4) $f(x)=\sqrt{\frac{x^3-4x}{x^2+x-6}}$ podmínky: $x^2+x-6\neq 0$

$(x+3)(x-2)\neq 0$

$\underline{x\neq -3}\ \wedge\ \underline{x\neq 2}$

$\wedge\ \frac{x^3-4x}{x^2+x-6}\geq 0$

$\frac{x^3-4x}{x^2+x-6}=\frac{x(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}=\frac{x(x+2)}{x+3}\overset{?}{\geq} 0$

x	−	−	−	+
$x+2$	−	−	+	+
$x+3$	−	+	+	+
$\frac{x(x+2)}{x+3}$	−	+	−	+

$-\infty$ $\quad -3 \quad -2 \quad 0 \quad +\infty$

$D_f=(-3,-2\rangle\cup\langle 0,2)\cup(2,\infty)$

$$g(x) = \frac{1}{1-\log_3(2-x)}$$

podmínky: I. $2-x>0$

$$\boxed{x<2}$$

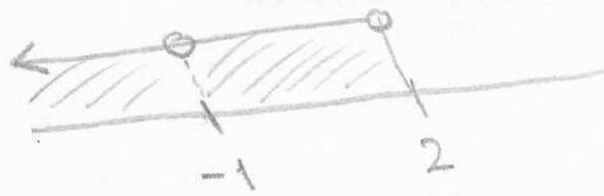
II. $1-\log_3(2-x) \neq 0$

$\log_3(2-x) \neq 1$

$2-x \neq 3^1$

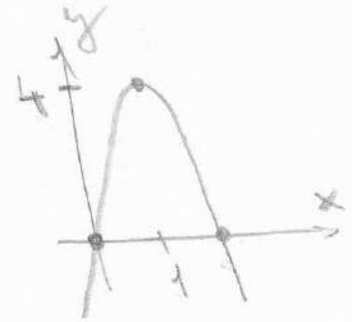
$-x \neq 1$

$$\boxed{x \neq -1}$$



$$D_g = (-\infty, 2) \setminus \{-1\}$$

5)



Vrcholová rovnice paraboly s vrcholem [1,4] je

$$y = A\cdot(x-1)^2+4$$

Víme, že parabola prochází bodem [0,0], proto

$$0 = A\cdot(0-1)^2+4$$

$$0 = A+4 \Rightarrow A=-4$$

tedy $\boxed{y = -4(x-1)^2+4 = -4x^2+8x}$

6) a) $\lim\limits_{x\to 2} \frac{\sqrt{x+7}-3}{4-x^2} \cdot \frac{\sqrt{x+7}+3}{\sqrt{x+7}+3} =$

$= \lim\limits_{x\to 2} \frac{(x+7)-9}{(4-x^2)(\sqrt{x+7}+3)} = \lim\limits_{x\to 2} \frac{\cancel{x-2}}{\underbrace{\cancel{(2-x)}}_{-1}(2+x)(\sqrt{x+7}+3)}$

$= \frac{1}{(-1)\cdot(2+2)\cdot(\sqrt{2+7}+3)} = \frac{1}{(-4)\cdot 6} = -\frac{1}{24}$

b) $\lim\limits_{x\to\infty} \frac{2^{x+3}+3^{x+2}}{2^{x+2}+3^{x+3}} = \lim\limits_{x\to\infty} \frac{8\cdot 2^x + 9\cdot 3^x}{4\cdot 2^x + 27\cdot 3^x} =$

$= \lim\limits_{x\to\infty} \frac{\cancel{3^x}\cdot\left(8\cdot\left(\frac{2}{3}\right)^x + 9\right)}{\cancel{3^x}\cdot\left(4\cdot\left(\frac{2}{3}\right)^x + 27\right)} = \frac{8\cdot 0+9}{4\cdot 0+27} = \frac{9}{27} = \frac{1}{3}$

$\to 0$

c) $\lim\limits_{x\to\infty}\left(\log_{\sqrt{9}}\left(\frac{6x^2+1}{2x^2+x+3}\right) + \sqrt{\frac{2x+1}{8x+1}}\right) =$

$\to 0$

$= \log_{\sqrt{9}}\left(\lim\limits_{x\to\infty} \frac{\cancel{x^2}\cdot\left(6+\frac{1}{x^2}\right)}{\cancel{x^2}\cdot\left(2+\frac{1}{x}+\frac{3}{x^2}\right)}\right) + \sqrt{\lim\limits_{x\to\infty}\frac{\cancel{x}\left(2+\frac{1}{x}\right)}{\cancel{x}\left(8+\frac{1}{x}\right)}}$

$0 \quad 0$

$= \log_{\sqrt{9}}\left(\frac{6+0}{2+0+0}\right) + \sqrt{\frac{2+0}{8+0}} = \log_{\sqrt{9}} 3 + \sqrt{\frac{1}{4}}$

$= \frac{1}{2} + \frac{1}{2} = 1$