(1.) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

$$\exists\, a, b \in \mathbb{R}: \quad \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \stackrel{?}{=} a \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + b \cdot \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{array}\right) \begin{array}{l} | \cdot (-3) \\ | \cdot (-2) \\ 2 \oplus \\ \oplus \end{array} \sim \left(\begin{array}{cc|c} 1 & 4 & 7 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{array}\right) \Big\} \Rightarrow b = 2$$

$$1a + 4 \cdot 2 = 7$$
$$a = -1$$

Ano, lze: $\vec{u}_3 = (-1)\,\vec{u}_1 + 2\vec{u}_2$

Vektory jsou lineárně závislé $\Rightarrow$ matice A je singulární

(2.) $B = \begin{pmatrix} 2 & 7 & -9 \\ 1 & -4 & 3 \\ 1 & -1 & 0 \end{pmatrix}$

$$\det(B - \lambda I) = \begin{vmatrix} 2-\lambda & 7 & -9 \\ 1 & -4-\lambda & 3 \\ 1 & -1 & -\lambda \end{vmatrix} = (2-\lambda)(-4-\lambda)(-\lambda) + 9 + 21 - \big((-9)(-4-\lambda) + 3(2-\lambda) - 7\lambda\big)$$

$$= (2-\lambda)(4\lambda + \lambda^2) + 30 - (36 + 9\lambda - 6 + 3\lambda - 7\lambda)$$

$$= 8\lambda - 4\lambda^2 + 2\lambda^2 - \lambda^3 + 30 - 30 - 5\lambda = -\lambda^3 - 2\lambda^2 + 3\lambda$$

$$= (-\lambda)(\lambda^2 + 2\lambda - 3)$$

$$= (-\lambda)(\lambda + 3)(\lambda - 1)$$

$\underline{\lambda = 0}$: $\begin{pmatrix} 2 & 7 & -9 \\ 1 & -4 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ $\quad \vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\underline{\lambda = 1}$: $\begin{pmatrix} 1 & 7 & -9 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{pmatrix}$ $\quad \vec{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$\underline{\lambda = -3}$: $\begin{pmatrix} 5 & 7 & -9 \\ 1 & -1 & 3 \\ 1 & -1 & 3 \end{pmatrix}$ $\quad \vec{x} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

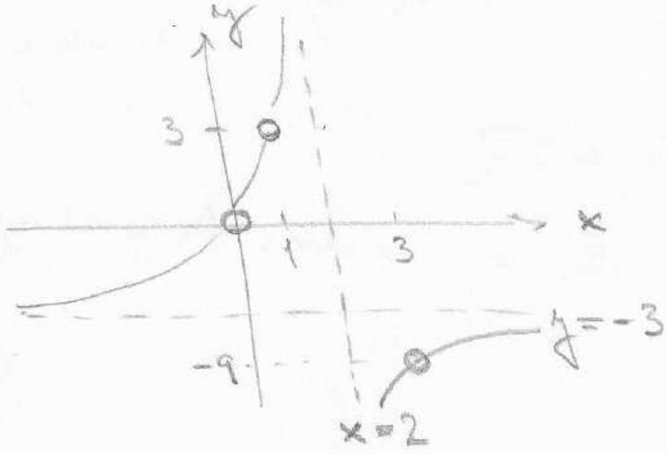
(3.) $f(x) = \left(\frac{3}{x-2} - \frac{6}{x-1} \right) : \left(\frac{1}{x} - \frac{2}{x^2-x} \right)$

$$= \frac{3(x-1)-6(x-2)}{(x-2)(x-1)} : \frac{x-1-2}{x(x-1)} = \frac{-3x+9}{(x-2)\cancel{(x-1)}} \cdot \frac{x\cancel{(x-1)}}{x-3} =$$

$$= \frac{(-3)\cancel{(x-3)}\cdot x}{(x-2)\cancel{(x-3)}} = -\frac{3x}{x-2}$$

$D_f = \mathbb{R} \setminus \{0;1;2;3\}$

$H_f = \mathbb{R} \setminus \{0;3;-3;-9\}$

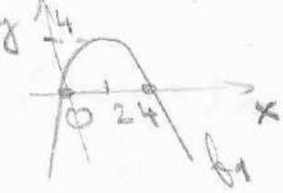


(4.) $f(x) = \log_{\frac{1}{8}}(4x - x^2) = (f_2 \circ f_1)(x)$

$4x - x^2 > 0$

$x(4-x) > 0$

$x \in (0,4)$



$D_f = (0,4) \xrightarrow{f_1} (0,4\rangle \xrightarrow{f_2} \langle -\frac{2}{3}, +\infty)$.

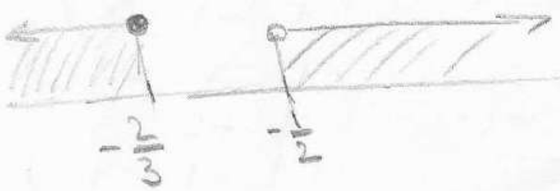
$f(1) = f(3) \Rightarrow f$ není prostá

$g(x) = \arcsin\left(\frac{x+1}{2x+1}\right)$

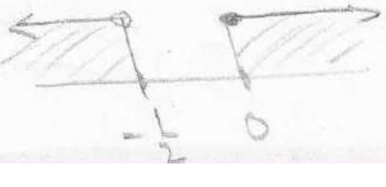
I. $\frac{x+1}{2x+1} \leq 1$ $\wedge$ II. $-1 \leq \frac{x+1}{2x+1}$

$\frac{x+1-(2x+1)}{2x+1} = -\frac{x}{2x+1} \leq 0$

$0 \leq \frac{x+1+2x+1}{2x+1} = \frac{3x+2}{2x+1}$



$\frac{x}{2x+1} \geq 0$



$D_g = (-\infty, -\frac{2}{3}\rangle \cup \langle 0, \infty)$

$4g)^*$ $g: y = \arcsin\left(\frac{x+1}{2x+1}\right)$

$$\sin y = \frac{x+1}{2x+1}$$

$$(2x+1)\sin y = x+1$$

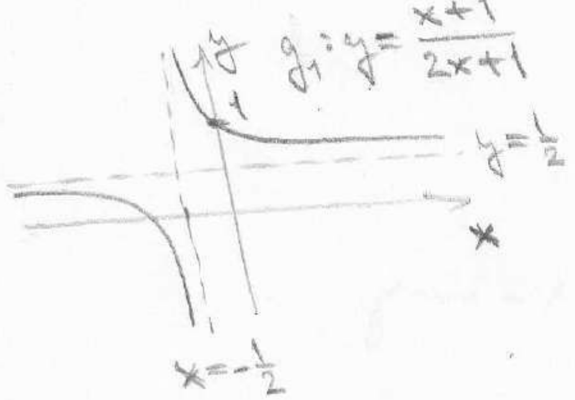
$$2x\sin y - x = 1 - \sin y$$

$$(2\sin y - 1)\cdot x = 1 - \sin y$$

$$x = \frac{1-\sin y}{2\sin y - 1}$$

$$g^{-1}: y = \frac{1-\sin x}{2\sin x - 1} \quad , x \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle \setminus \left\{\frac{\pi}{6}\right\}$$

$$D_g = (-\infty, -\tfrac{2}{3}\rangle \cup \langle 0, \infty) \xrightarrow{g_1} \langle -1, 1\rangle \setminus \left\{\tfrac{1}{2}\right\} \xrightarrow{g_2} \left\langle -\tfrac{\pi}{2}, \tfrac{\pi}{2}\right\rangle \setminus \left\{\tfrac{\pi}{6}\right\}$$



$$\arcsin \frac{1}{2} = \frac{\pi}{6}$$

$$H_g = \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle \setminus \left\{\frac{\pi}{6}\right\}$$

5.
$$\lim_{x\to 1} \frac{4^x + 2^{x+2} - 12}{4^x - 2^{x+2} + 4} = \lim_{x\to 1} \frac{(2^x)^2 + 4\cdot 2^x - 12}{(2^x)^2 - 4\cdot 2^x + 4} =$$

$$= \lim_{x\to 1} \frac{\cancel{(2^x - 2)}(2^x + 6)}{\cancel{(2^x - 2)}(2^x - 2)} = \lim_{x\to 1} \frac{2^x + 6}{2^x - 2}$$ limita neexistuje

lebo $\lim_{x\to 1_+}(\;) = \frac{8}{0_+} = +\infty$

a $\lim_{x\to 1_-}(\;) = \frac{8}{0_-} = -\infty$

$$\lim_{x\to +\infty} \frac{2^{3x-1} + (\sqrt{3})^{4x+1}}{2^{3x+1} + (\sqrt{3})^{4x-3}} = \lim_{x\to\infty} \frac{\frac{1}{2}\cdot 8^x + \sqrt{3}\cdot 9^x}{2\cdot 8^x + (\sqrt{3})^{-3}\cdot 9^x} =$$

$$= \lim_{x\to\infty} \frac{\cancel{9^x}\left(\frac{1}{2}\cdot\left(\frac{8}{9}\right)^x + \sqrt{3}\right)}{\cancel{9^x}\left(2\cdot\left(\frac{8}{9}\right)^x + (\sqrt{3})^{-3}\right)} = \frac{0 + \sqrt{3}}{0 + (\sqrt{3})^{-3}} = (\sqrt{3})^4 = \underline{\underline{9}}$$

$$\lim_{x\to -\infty} \left(\sqrt{x^2+2x} + x\right) \cdot \frac{\sqrt{x^2+2x} - x}{\sqrt{x^2+2x} - x} = \lim_{x\to -\infty} \frac{x^2 + 2x - x^2}{|x|\cdot\sqrt{1+\frac{2}{x}} - x} =$$

$$\overset{x<0}{=} \lim_{x\to -\infty} \frac{2x}{-x\cdot\sqrt{1+\frac{2}{x}} - x} = \lim_{x\to\infty} \frac{\cancel{x}\cdot 2}{\cancel{x}\cdot\left(-\sqrt{1+\frac{2}{x}} - 1\right)} =$$

$$= \frac{2}{-1-1} = \underline{\underline{-1}}$$