

1.

$$\vec{\mu}_1 = (2, 1, -1)$$

$$\vec{\mu}_2 = (1, 3, 1)$$

$$\vec{\mu}_3 = (6, -2, -6)$$

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ 6 & -2 & -6 \end{pmatrix} \begin{array}{l} \cdot (-2) \\ \oplus \\ \oplus \\ \oplus \end{array} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & -5 & -3 \\ 0 & -20 & -12 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & -5 & -3 \\ 0 & -20 & -12 \end{pmatrix} \begin{array}{l} \cdot (-1) \\ | : 4 \\ \oplus \end{array} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & -5 & -3 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow \vec{\mu}_1, \vec{\mu}_2, \vec{\mu}_3$ jsou lineárně závislé

$$\exists c_1, c_2 \in \mathbb{R} : \vec{\mu}_3 = c_1 \vec{\mu}_1 + c_2 \vec{\mu}_2$$

$$\begin{pmatrix} 6 & 2 & 1 \\ -2 & 1 & 3 \\ -6 & -1 & 1 \end{pmatrix} \begin{array}{l} \oplus \\ \oplus \end{array} \sim \begin{pmatrix} 6 & 2 & 1 \\ -2 & 1 & 3 \\ -8 & 0 & 4 \end{pmatrix} \Rightarrow c_2 = -2$$

$$6 = 2c_1 + c_2$$

$$6 = 2 \cdot 4 - 2 \quad \checkmark$$

Ans, lze.

$$\vec{\mu}_3 = 4\vec{\mu}_1 + (-2)\vec{\mu}_2$$

$$\begin{pmatrix} 6 \\ -2 \\ -6 \end{pmatrix} = 4 \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + (-2) \cdot \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

2.

$$A \cdot A^T = \begin{pmatrix} 2 & 1 & 0 & -1 \\ 3 & 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 1 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 2 & 26 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \cdot (-3) \\ \oplus \\ \oplus \end{array}$$

3.

$$\begin{pmatrix} -2 & 0 & a \\ a & 3 & -8 \\ 0 & 1 & a \end{pmatrix} \begin{matrix} | \cdot a \neq 0 \\ | \cdot 2 \end{matrix} \sim \begin{pmatrix} -2a & 0 & a^2 \\ 2a & 6 & -16 \\ 0 & 1 & a \end{pmatrix} \rightarrow \oplus$$

$$\sim \begin{pmatrix} -2 & 0 & a \\ 0 & 6 & a^2 - 16 \\ 0 & 1 & a \end{pmatrix} \begin{matrix} | \cdot (-6) \\ | \cdot (-6) \end{matrix} \xrightarrow{\oplus} \sim \begin{pmatrix} -2 & 0 & a \\ 0 & 6 & a^2 - 16 \\ 0 & 0 & a^2 - 6a - 16 \end{pmatrix}$$

Matice nemá plnou hodnost,

pokud $a^2 - 6a - 16 = 0$

$$(a-8)(a+2) = 0$$

$$\underline{a=8} \vee \underline{a=-2}$$

$\forall a \in \mathbb{R} \setminus \{8; -2\}$: hodnost je 3

$$\underline{a=8} : \begin{pmatrix} -2 & 0 & 8 \\ 0 & 6 & 48 \end{pmatrix}$$

$$\underline{a=-2} : \begin{pmatrix} -2 & 0 & -2 \\ 0 & 6 & -12 \end{pmatrix}$$

} hodnost těchto matic je 2.

LOKHOLA	8402/1/29
ZEJALOLA	8103/1/208
BOJAKOVS	8702/1/788
PETERKVT	8402/1/288
MEGEROLA	8402/1/228
WUZKI	8402/1/228
KOPINKOG	8402/1/228
KOCNICHV	8103/1/208
TIVSEKOE	8402/1/228
TOGIKON	8402/1/228
EOKTEBO	8103/1/208
FOUSEK	8103/1/208
DAOBAKV	8103/1/208
ONIDOLAE	8103/1/208
SIGEV	8402/1/228
LOGIN	0001