

1a)  $f(x,y) = \ln(x^2 + xy^3)$

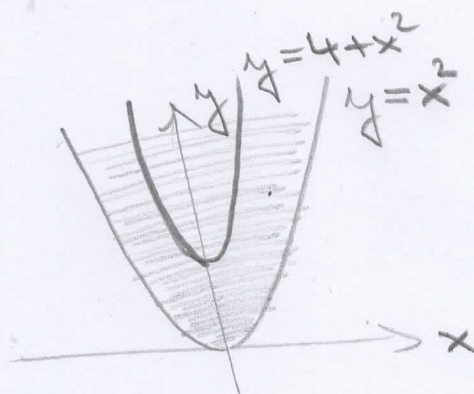
$$\frac{\partial f}{\partial x} = \frac{2x + y^3}{x^2 + xy^3}$$

$$\frac{\partial f}{\partial y} = \frac{3xy^2}{x^2 + xy^3}$$

1b)  $f(x,y) = x^y$

$$\frac{\partial f}{\partial x} = yx^{y-1}$$

$$\frac{\partial f}{\partial y} = x^y \cdot \ln x$$



2)  $f(x,y) = \sqrt{y-x^2}$

$$y - x^2 \geq 0$$

$$y \geq x^2$$

$$f(1,5) = \sqrt{5-1^2} = \sqrt{4} = 2$$

rovnice vlnice:

$$f(x,y) = f(1,5)$$

$$\sqrt{y-x^2} = 2$$

$$y-x^2 = 4$$

$$y = 4 + x^2$$

3)  $f(x,y) = \arcsin(x^2 + y^2 + 6x + 4) + \sqrt{xy}$

$$-1 \leq x^2 + y^2 + 6x + 4 \leq 1$$

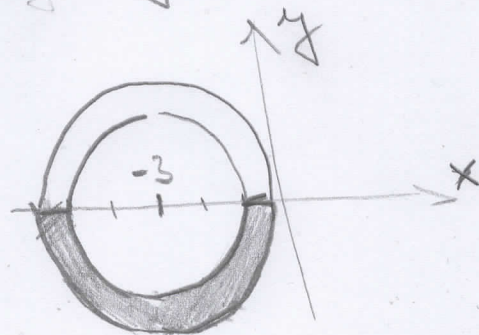
$$\wedge xy \geq 0 \quad (\text{1. a 3. kvadrant})$$

$$-1 \leq x^2 + 6x + 9 - 9 + 4 + y^2 \leq 1$$

$$-1 \leq (x+3)^2 + y^2 - 5 \leq 1 \quad | +5$$

$$4 \leq (x+3)^2 + y^2 \leq 6$$

mezikruží mezi kružnicemi se středem v bodě  $[-3,0]$  a poloměry 2 a  $\sqrt{6}$



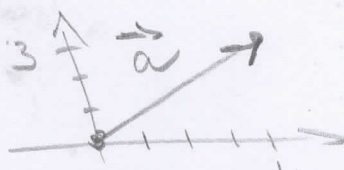
$$4. \quad f(x,y) = x^3 + y^2 + xy$$

$$\frac{\partial f}{\partial x} = 3x^2 + y \quad \left| \begin{array}{l} [1,2] \\ = 5 \end{array} \right. \quad \left. \vphantom{\frac{\partial f}{\partial x}} \right\} \text{grad} f(1,2) = (5,5)$$

$$\frac{\partial f}{\partial y} = 2y + x \quad \left| \begin{array}{l} [1,2] \\ = 5 \end{array} \right.$$

$$\vec{a} = (4,3)$$

$$\|\vec{a}\| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$



směrová derivace funkce  $f$  v bodě  $[1,2]$   
ve směru  $\vec{a}$  je:

$$\text{grad} f(1,2) \cdot \frac{\vec{a}}{\|\vec{a}\|} = (5,5) \cdot \left( \frac{4}{5}, \frac{3}{5} \right) =$$

$$= 5 \cdot \frac{4}{5} + 5 \cdot \frac{3}{5} = 4 + 3 = \underline{\underline{7}}$$