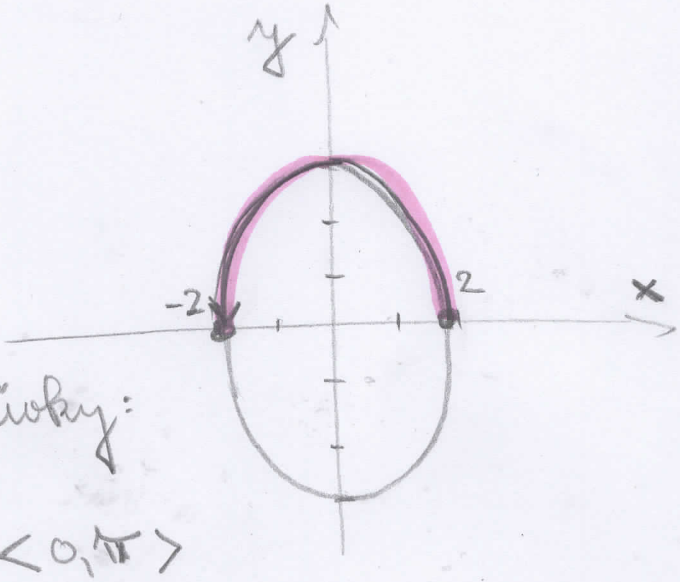
1. $F(x,y) = (y^2, x^2)$



$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

parametrické vyjádření křivky:

$$\left| \begin{array}{l} x = 2\cos t \\ y = 3\sin t \end{array} \right| \quad t \in \langle 0, \pi \rangle$$

$dx = -2\sin t \, dt$
$dy = 3\cos t \, dt$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi} (9\sin^2 t, 4\cos^2 t) \cdot (-2\sin t, 3\cos t) \, dt$$

⌈ Je-li $F: \mathbb{R}^2 \to \mathbb{R}^2$ vektorové pole
a $\varphi: [a,b] \to \mathbb{R}^2$ je hladká křivka,
pak $\boxed{\int_\varphi \vec{F} \cdot d\vec{r} = \int_a^b (F \circ \varphi)(t) \cdot \varphi'(t) \, dt}$ ⌋

$$= \int_0^{\pi} (-18\sin^3 t + 12\cos^3 t) \, dt$$

$$= -18 \int_0^{\pi} \sin t \cdot (1 - \cos^2 t) \, dt + 12 \int_0^{\pi} \cos t \, (1 - \sin^2 t) \, dt$$

$$\left| \begin{array}{l} \cos t = a \\ -\sin t \, dt = da \end{array} \right| \qquad \left| \begin{array}{l} \sin t = a \\ \cos t \, dt = da \end{array} \right|$$

$$= 18 \int_1^{-1} (1 - a^2) \, da + 12 \int_0^0 (1 - a^2) \, da = 18 \cdot \left[a - \frac{a^3}{3} \right]_1^{-1} = \underline{\underline{-24}}$$

(2.) $F(x,y,z) = (x-z, 1-xy, y)$

$$\gamma: \begin{array}{l} x = t \\ y = t^2 \\ z = t^3 \end{array} \quad \begin{array}{l} dx = dt \\ dy = 2t\,dt \\ dz = 3t^2\,dt \end{array}$$

$t \in \langle 0,1 \rangle$

$$\int_\gamma \vec{F} \cdot d\vec{r} = \int_0^1 \left(t - t^3, 1 - t\cdot t^2, t^2\right)\cdot\left(1, 2t, 3t^2\right) dt$$

$$= \int_0^1 \left(t - t^3 + 2t - 2t^4 + 3t^4\right) dt$$

$$= \int_0^1 \left(t^4 + 3t - t^3\right) dt = \left[\frac{t^5}{5} + 3\frac{t^2}{2} - \frac{t^4}{4}\right]_0^1$$

$$= \frac{1}{5} + \frac{3}{2} - \frac{1}{4} = \frac{4+30-5}{20} = \frac{29}{20}$$