

## 2. minitest LA2

26. 3. 2025

Najděte vlastní čísla a vlastní vektory matice

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

Charakteristický polynom:

$$f(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 4 = \\ = (3-\lambda-2)(3-\lambda+2) = (1-\lambda)(5-\lambda)$$

$$f(\lambda) = 0 \iff \underline{\lambda=1} \vee \underline{\lambda=5}$$

Vlastní vektory:

$$\underline{\lambda_1=1}: (A - 1 \cdot I) \vec{x} = \vec{0} \\ \left( \begin{array}{cc|c} 2 & 2 & 0 \\ 2 & 2 & 0 \end{array} \right) \quad \vec{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\underline{\lambda_2=5}: (A - 5I) \vec{x} = \vec{0} \\ \left( \begin{array}{cc|c} -2 & 2 & 0 \\ 2 & -2 & 0 \end{array} \right) \quad \vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

diagonalizace:  $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1}$