6. minitest MA1

Varianta A

18. 12. 2024

1) Z definice derivace vypočtěte 1. derivaci funkce

$$f(x) = \ln^2 x$$

v bodě $x = 3$.

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{\ln^2 x - \ln^2 3}{x - 3} =$$

$$= \lim_{x \to 3} \frac{(\ln x - \ln 3)(\ln x + \ln 3)}{x - 3} \overset{\text{VOAL}}{=} \underbrace{\lim_{x \to 3} (\ln x + \ln 3)}_{= 2 \cdot \ln 3} \cdot \lim_{x \to 3} \frac{\ln \frac{x}{3}}{x - 3}$$

$$= 2 \ln 3 \cdot \frac{1}{3} \cdot \underbrace{\lim_{x \to 3} \frac{\ln \frac{x}{3}}{\frac{x}{3} - 1}}_{\underset{\text{VOLSF}}{=} 1} = \frac{2 \ln 3}{3}$$

2) Vypočtěte limitu funkce

$$\lim_{x\to 0}\left(\frac{\operatorname{arccotg} x+\arcsin x}{\operatorname{arccotg} x-\arcsin x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0} e^{\frac{1}{x}\cdot\ln\left(\frac{\operatorname{arccotg} x+\arcsin x}{\operatorname{arccotg} x-\arcsin x}\right)} \overset{\text{VOLSF}}{=} e^{\lim\limits_{x\to 0}(\dots)} \overset{(*)}{=} \underline{\underline{e^{\frac{4}{\pi}}}}$$

$$\lim_{x\to 0}\frac{1}{x}\cdot\ln\left(\frac{\operatorname{arccotg} x+\arcsin x}{\operatorname{arccotg} x-\arcsin x}\right)\cdot\frac{\frac{\operatorname{arccotg} x+\arcsin x}{\operatorname{arccotg} x-\arcsin x}-1}{\frac{\operatorname{arccotg} x+\arcsin x}{\operatorname{arccotg} x-\arcsin x}-1}$$

$$\overset{\text{VOAL}}{=} \underbrace{\lim_{x\to 0}\frac{\ln\left(\frac{\operatorname{arccotg} x+\arcsin x}{\operatorname{arccotg} x-\arcsin x}\right)}{\frac{\operatorname{arccotg} x+\arcsin x}{\operatorname{arccotg} x-\arcsin x}-1}}_{\overset{\text{VOLSF}}{=} 1} \cdot \lim_{x\to 0}\frac{1}{x}\cdot\frac{2\arcsin x}{\operatorname{arccotg} x-\arcsin x}$$

podmínka (P)

$$= 2\cdot\underbrace{\lim_{x\to 0}\frac{\arcsin x}{x}}_{=1}\cdot\underbrace{\lim_{x\to 0}\frac{1}{\operatorname{arccotg} x-\arcsin x}}_{\frac{1}{\frac{\pi}{2}-0}}$$

$$= 2\cdot\frac{2}{\pi} = \frac{4}{\pi}$$

6. minitest MA1

Varianta B

18. 12. 2024

1) Z definice derivace vypočtěte 1. derivaci funkce

$$f(x) = \sqrt{4x^2 + 5}$$

v bodě $x = 1$.

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{\sqrt{4x^2+5} - 3}{x - 1} \cdot \frac{\sqrt{4x^2+5} + 3}{\sqrt{4x^2+5} + 3} =$$

$$= \lim_{x \to 1} \frac{4x^2 + 5 - 9}{(x-1)\left(\sqrt{4x^2+5} + 3\right)} \overset{\text{VOAL}}{=} \underbrace{\lim_{x \to 1} \frac{1}{\sqrt{4x^2+5} + 3}}_{= \frac{1}{\sqrt{9}+3} = \frac{1}{6}} \cdot \lim_{x \to 1} \frac{4x^2 - 4}{x - 1}$$

$$= \frac{1}{6} \cdot 4 \cdot \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = \frac{4}{6} \cdot 2 = \underline{\underline{\frac{4}{3}}}$$

2) Vypočtěte limitu funkce.

$$\lim_{x\to\infty}\left(\frac{1+\operatorname{arccotg} x}{1-\operatorname{arccotg} x}\right)^x$$

$$= \lim_{x\to\infty} e^{x\ln\left(\frac{1+\operatorname{arccotg}x}{1-\operatorname{arccotg}x}\right)} \overset{\text{VOLSF}}{=} e^{\lim_{x\to\infty}(\;)} = \dots = \underline{\underline{e^2}}$$

$$\lim_{x\to\infty} x\cdot\ln\left(\frac{1+\operatorname{arccotg}x}{1-\operatorname{arccotg}x}\right)\cdot\frac{\frac{1+\operatorname{arccotg}x}{1-\operatorname{arccotg}x}-1}{\frac{1+\operatorname{arccotg}x}{1-\operatorname{arccotg}x}-1} =$$

$$\overset{\text{VOAL}}{=} \underbrace{\lim_{x\to\infty}\frac{\ln\left(\frac{1+\operatorname{arccotg}x}{1-\operatorname{arccotg}x}\right)}{\frac{1+\operatorname{arccotg}x}{1-\operatorname{arccotg}x}-1}}_{\overset{\text{VOLSF}}{=}1}\cdot\lim_{x\to\infty} x\cdot\frac{2\operatorname{arccotg}x}{1-\operatorname{arccotg}x}$$

(podmínka (P))

$$= 2\cdot\underbrace{\lim_{x\to\infty}\frac{1}{1-\operatorname{arccotg}x}}_{=\frac{1}{1-0}=1}\cdot\lim_{x\to\infty} x\operatorname{arccotg}x =$$

$$= 2\cdot\lim_{y\to 0_+}\operatorname{cotg}y\cdot y = 2\underbrace{\lim_{y\to 0_+}\cos y}_{1}\cdot\underbrace{\lim_{y\to 0_+}\frac{y}{\sin y}}_{1} = 2$$

$$\left|x=\operatorname{cotg}y\right|_{y\in(0,\pi)}$$