

12. minitest MAT2

2. 4. 2025

Vypočtete délku části grafu funkce

$$f(x) = \sqrt{x^3}$$

na intervalu $[0, \frac{4}{3}]$.

$$\int_0^{\frac{4}{3}} \sqrt{1 + \left(\sqrt{x^3}\right)'}^2 dx = \int_0^{\frac{4}{3}} \sqrt{1 + \frac{9}{4}x} dx =$$

$\left(\sqrt{x^3}\right)' = \left(x^{\frac{3}{2}}\right)' = \frac{3}{2}x^{\frac{1}{2}}$

$$= \int_1^4 \sqrt{t} \cdot \frac{4}{9} dt = \frac{4}{9} \cdot \left[\frac{2}{3} t^{\frac{3}{2}} \right]_1^4 =$$

substituce

$$\left| \begin{array}{l} 1 + \frac{9}{4}x = t \\ \frac{9}{4}dx = dt \end{array} \right| \Rightarrow dx = \frac{4}{9}dt$$

$$= \frac{4}{9} \cdot \frac{2}{3} \cdot \left(4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{8}{27} \cdot \underbrace{(8-1)}_7 = \underline{\underline{\frac{56}{27}}}$$