

16. minitest MAT2

17. 4. 2025

Vypočítejte limitu posloupnosti

$$\lim_{n \rightarrow \infty} \left(\frac{3n^2 + n + 1}{3n^2 - n + 1} \right)^n$$

$$\frac{3n^2 + n + 1}{3n^2 - n + 1} = \frac{3n^2 - n + 1}{3n^2 - n + 1} + \frac{2n}{3n^2 - n + 1} = 1 + \frac{1}{\frac{3n^2 - n + 1}{2n}}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(\frac{3n^2 + n + 1}{3n^2 - n + 1} \right)^n &= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\frac{3n^2 - n + 1}{2n}} \right)^n = \\ &= \lim_{n \rightarrow \infty} \left(\underbrace{\left(1 + \frac{1}{\frac{3n^2 - n + 1}{2n}} \right)^{\frac{3n^2 - n + 1}{2n}}}_{=: a_n} \right)^{\frac{2n^2}{3n^2 - n + 1}} = \lim_{n \rightarrow \infty} a_n^{\frac{2n^2}{3n^2 - n + 1}} \end{aligned}$$

$$= \left(\lim_{n \rightarrow \infty} a_n \right)^{\lim_{n \rightarrow \infty} \frac{2n^2}{3n^2 - n + 1}} = \underline{\underline{e^{\frac{2}{3}}}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2n^2}{3n^2 - n + 1} = \lim_{n \rightarrow \infty} \frac{2}{3 - \frac{1}{n} + \frac{1}{n^2}} = \frac{2}{3}$$