18. minitest

Vyšetřete konvergenci řady $\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}$

Podílové kritérium

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}\cdot(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\cdot n!}{n^n}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{3^n}\cdot 3^1}{\cancel{3^n}} \cdot \frac{\cancel{(n+1)\cdot n!}}{\cancel{n!}} \cdot \frac{n^n}{\cancel{(n+1)}\cdot(n+1)^n} =$$

$$= 3\cdot \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = 3\cdot \lim_{n\to\infty}\left(1+\frac{(-1)}{n+1}\right)^{n+1-1} =$$

$$= 3\cdot \underbrace{\lim_{n\to\infty}\left(1+\frac{(-1)}{n+1}\right)^{n+1}}_{=e^{-1}} \cdot \underbrace{\lim_{n\to\infty}\left(1+\frac{(-1)}{n+1}\right)^{-1}}_{=1} =$$

$$= \frac{3}{e} > 1 \implies \sum_{1}^{\infty} \frac{3^n\cdot n!}{n^n} \text{ DIVERGUJE}$$