1. minitest MAT2

Varianta B

20. 2. 2025

Vypočtěte integrál

$$\int \frac{x}{x^4-1}\,dx$$

$$\frac{x}{x^4-1} = \frac{x}{(x^2-1)(x^2+1)} = \frac{x}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

$$x = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+1)$$

$x=1$: $1 = 4A + 0B + 0(C+D) \Rightarrow \boxed{A = \frac{1}{4}}$

$x=-1$: $-1 = 0A - 4B + 0(-C+D) \Rightarrow \boxed{B = \frac{1}{4}}$

$x=i$: $i = 0A + 0B + ((Ci+D)(i^2-1)$

$i = -2Ci - 2D$ $\Rightarrow$ $\boxed{D=0}$

$-2C = 1$

$\boxed{C = -\frac{1}{2}}$

$$\int \frac{x}{x^4-1}\,dx = \int \left(\frac{\frac{1}{4}}{x-1} + \frac{\frac{1}{4}}{x+1} - \frac{\frac{1}{2}x}{x^2+1} \right) dx =$$

$$= \frac{1}{4}\ln|x-1| + \frac{1}{4}\ln|x+1| - \frac{1}{2}\int \frac{x}{x^2+1}\,dx$$

$$= \frac{1}{4}\ln\left|\frac{x^2-1}{x^2+1}\right| + C, \quad C \in \mathbb{R}$$

$\left| \begin{matrix} x^2+1 = t \\ 2x\,dx = dt \end{matrix} \right| = \frac{1}{4}\int \frac{1}{t}\,dt = \frac{1}{4}\ln|t|$

1. minitest MAT2

Varianta A

20. 2. 2025

Vypočtěte integrál

$$\int \frac{x^2}{x^2 - 3x + 2}\, dx$$

$x^2 : (x^2 - 3x + 2) = 1 + \dfrac{3x-2}{x^2-3x+2}$

$-(x^2 - 3x + 2)$

$3x - 2$

$$\frac{3x-2}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1} \quad | \cdot (x-2)(x-1)$$

$$3x - 2 = A(x-1) + B(x-2)$$

$x=1$: $\quad 1 = 0A - 1B \Rightarrow \boxed{B = -1}$

$x=2$: $\quad 4 = 1A + 0B \Rightarrow \boxed{A = 4}$

$$\int \frac{x^2}{x^2-3x+2}\, dx = \int \left(1 + \frac{4}{x-2} - \frac{1}{x-1}\right) dx =$$

$$= x + 4\ln|x-2| - \ln|x-1| + C, \quad C \in \mathbb{R}$$