# 2. minitest MAT2

Varianta A

26. 2. 2025

Vypočtěte integrál

$$\int \sin^4 x \cos^3 x \, dx$$

$$\int \sin^4 x \underbrace{\cos^3 x}_{\cos^2 x \cdot \cos x} \, dx = \int \sin^4 x \cdot (1 - \sin^2 x) \cos x \, dx =$$

$$\underset{\left| \begin{array}{l} \sin x = t \\ \cos x \, dx = dt \end{array} \right|}{=} \int t^4 (1 - t^2) \, dt = \int (t^4 - t^6) \, dt = \frac{t^5}{5} - \frac{t^7}{7} + c$$

$$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c, \; c \in \mathbb{R}$$

# 2. minitest MAT2

Varianta B

26. 2. 2025

Vypočtěte integrál

$$\int \frac{1}{\cos x}\, dx$$

$$\int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx = \int \frac{\cos x}{1-\sin^2 x} dx = \int \frac{1}{1-t^2} dt$$

$$\left| \begin{array}{l} \sin x = t \\ \cos x\, dx = dt \end{array} \right|$$

$$\overset{(*)}{=} \frac{1}{2}\int \left(\frac{1}{1-t} + \frac{1}{1+t}\right) dt = \frac{1}{2}\left(-\ln|1-t| + \ln|1+t|\right) + C = \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + c$$

$$\frac{1}{1-t^2} = \frac{1}{(1-t)(1+t)} = \frac{A}{1-t} + \frac{B}{1+t} \quad \Big| \cdot (1-t)(1+t)$$

$$1 = A(1+t) + B(1-t)$$

$t=1$: $\quad 1 = 2A + 0B \Rightarrow \boxed{A = \frac{1}{2}}$

$t=-1$: $\quad 1 = 0A + 2B \Rightarrow \boxed{B = \frac{1}{2}}$

Alternativně:

$$\int \frac{1}{\cos x}\,dx = \int \frac{1}{\frac{1-t^2}{1+t^2}} \cdot \frac{2}{t^2+1}\,dt = 2 \cdot \int \frac{1}{1-t^2}\,dt =$$

$$\overset{(*)}{=} \ln\left|\frac{1+t}{1-t}\right| + c$$

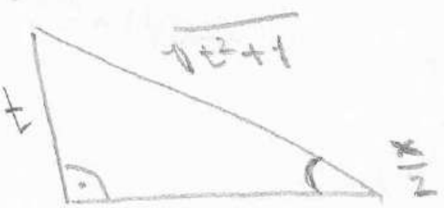
$$= \ln\left|\frac{1+\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}\frac{x}{2}}\right| + c$$

substituce $\left| t = \operatorname{tg}\frac{x}{2} \right|$

$$\operatorname{arctg} t = \frac{x}{2}$$

$$2\operatorname{arctg} t = x$$

$$\left| \frac{2}{t^2+1}\,dt = dx \right|$$




$$\sin\frac{x}{2} = \frac{t}{\sqrt{t^2+1}}$$

$$\cos\frac{x}{2} = \frac{1}{\sqrt{t^2+1}}$$

$$\sin x = \sin\left(2\cdot\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2} = \frac{2t}{t^2+1}$$

$$\cos x = \cos\left(2\cdot\frac{x}{2}\right) = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \frac{1-t^2}{t^2+1}$$