3. minitest MAT2

Varianta A

27. 2. 2025

Vypočtěte integrál

$$\int_1^\infty \frac{1}{x \ln^2 x} \, dx$$

$$\int_1^\infty \frac{1}{x \ln^2 x} dx = \int_0^\infty \frac{1}{t^2} dt = \left[-\frac{1}{t}\right]_0^\infty =$$

$$\left| \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \end{array} \right|$$

$$= \underbrace{\lim_{x \to +\infty} \left(-\frac{1}{t}\right)}_{0} - \underbrace{\lim_{x \to 0_+} \left(-\frac{1}{t}\right)}_{-\infty} = \underline{\underline{+\infty}}$$

3. minitest MAT2

Varianta B

27. 2. 2025

Vypočtěte integrál

$$\int_1^\infty \frac{1}{x^2+x}\,dx$$

$$\frac{1}{x^2+x} = \frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} \quad \Big| \cdot x(x+1)$$

$$1 = A(x+1) + Bx$$

$$\underline{x=0}: \quad 1 = A + 0B \quad \Rightarrow \boxed{A=1}$$

$$\underline{x=-1}: \quad 1 = 0A - B \quad \Rightarrow \boxed{B=-1}$$

$$\int_1^\infty \frac{1}{x^2+x}\,dx = \int_1^\infty \left(\frac{1}{x} - \frac{1}{x+1}\right) dx = \Big[\ln|x| - \ln|x+1|\Big]_1^\infty$$

$$= \left[\ln\left|\frac{x}{x+1}\right|\right]_1^\infty = \underbrace{\lim_{x\to\infty} \ln\left(\frac{x}{x+1}\right)}_{} - \ln\frac{1}{2}$$

$$= \underbrace{\ln 1}_{0} - \ln\frac{1}{2} = \underline{\underline{\ln 2}} \qquad \overset{\text{VOLSE}}{=} \ln\Big(\underbrace{\lim_{x\to\infty}\frac{x}{x+1}}_{\overset{\text{L.P.}}{=} \lim \frac{1}{1} = 1}\Big)$$