4. minitest MAT2

Varianta A

3. 3. 2025

Vypočtěte integrál

$$\int_1^\infty \frac{1}{x^3+6x^2+9x}\,dx$$

Rozklad na parciální zlomky:

$$\frac{1}{x^3+6x^2+9x} = \frac{1}{x(x^2+6x+9)} = \frac{1}{x(x+3)^2} = \frac{\overset{=\frac{1}{9}}{A}}{x} + \frac{\overset{=-\frac{1}{9}}{B}}{x+3} + \frac{\overset{=-\frac{1}{3}}{C}}{(x+3)^2}$$

$$1 = A(x+3)^2 + Bx(x+3) + Cx$$

$\underline{x=0}$: $1 = 9A \implies \boxed{A = \frac{1}{9}}$

$\underline{x=-3}$: $1 = -3C \implies \boxed{C = -\frac{1}{3}}$

$$1 = \frac{1}{9}\cdot(x^2+6x+9) + Bx^2 + 3Bx - \frac{1}{3}x$$

$$1 = \frac{1}{9}x^2 + \frac{2}{3}x + 1 + Bx^2 + 3Bx - \frac{1}{3}x$$

$$\implies 0 = \frac{1}{9} + B \implies \boxed{B = -\frac{1}{9}}$$

$$\int_1^\infty \left(\frac{\frac{1}{9}}{x} - \frac{\frac{1}{9}}{x+3} - \frac{\frac{1}{3}}{(x+3)^2}\right)dx = \left[\frac{1}{9}\ln|x| - \frac{1}{9}\ln|x+3| + \frac{1}{3}\cdot\frac{1}{x+3}\right]_1^\infty$$

$$= \lim_{x\to\infty}\left(\frac{1}{9}\ln\left|\frac{x}{x+3}\right| + \frac{1}{3}\cdot\underbrace{\frac{1}{x+3}}_{0}\right) - \left(\frac{1}{9}\underbrace{\ln 1}_{0} - \frac{1}{9}\ln 4 + \frac{1}{12}\right)$$

$$= \frac{1}{9}\ln\Big(\underbrace{\lim_{x\to\infty}\frac{x}{x+3}}_{1}\Big) + 0 + \frac{1}{9}\ln 4 - \frac{1}{12} = \underline{\underline{\frac{1}{9}\ln 4 - \frac{1}{12}}}$$

Varianta B

$$\int_1^{\infty} \frac{x^2}{(x^3+1)^2}\,dx = \frac{1}{3}\cdot\int_2^{\infty} \frac{dt}{t^2} = \frac{1}{3}\cdot\left[-\frac{1}{t}\right]_2^{\infty} =$$

substituce

$$\left| \begin{array}{l} x^3+1 = t \\ 3x^2\,dx = dt \end{array} \right|$$

$$x^2\,dx = \frac{1}{3}\,dt$$

$$= \frac{1}{3}\cdot\left(\underbrace{\lim_{t\to+\infty}\left(-\frac{1}{t}\right)}_{0} - \left(-\frac{1}{2}\right) \right) = \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6}$$