

$$1. \quad A = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 3 \\ 3 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 - 9 = (4-\lambda-3)(4-\lambda+3) \\ = (1-\lambda)(7-\lambda).$$

$$\lambda_1 = 1: (A - 1 \cdot I) \vec{x} = \vec{0}$$

$$\begin{pmatrix} 3 & 3 & | & 0 \\ 3 & 3 & | & 0 \end{pmatrix}$$

$$\text{Elastní vektor } \vec{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_2 = 7: (A - 7I) \vec{x} = \vec{0}$$

$$\begin{pmatrix} -3 & 3 & | & 0 \\ 3 & -3 & | & 0 \end{pmatrix}$$

$$\text{Elastní vektor } \vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{\det A} \cdot \text{adj} A = \frac{1}{16-9} \cdot \begin{pmatrix} 4 & -3 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & -\frac{3}{7} \\ -\frac{3}{7} & \frac{4}{7} \end{pmatrix}$$

$$2. \quad c_1 \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \\ -10 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 3 & 2 & 18 \\ -4 & 2 & -10 \end{array} \right) \begin{matrix} \oplus \\ \cdot (-3) \\ \cdot (4) \end{matrix} \sim \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 5 & 15 \\ 0 & -2 & -6 \end{array} \right) \begin{matrix} \\ :5 \\ :(-2) \end{matrix} \sim \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & 3 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array} \right)$$

\Rightarrow vektory jsou lin. závislé

$$\underline{c_2 = 3}$$

$$c_1 - c_2 = 1$$

$$\underline{c_1 = 4}$$

$$\vec{u}_3 = 4 \cdot \vec{u}_1 + 3 \cdot \vec{u}_2$$