11. minitest RMF

Varianta A

13. 12. 2024

Nalezněte řešení integro-diferenciální rovnice

$$y' - 6y + 5\int_0^x y(t)dt = 1$$

s počáteční podmínkou $y(0) = 0$.

$$\mathcal{L}\left(y' - 6y + 5\cdot\int_0^x y(t)\,dt\right) = \mathcal{L}(1)$$

$$\underbrace{\mathcal{L}(y')}_{p\cdot\mathcal{L}(y) - \underbrace{y(0_+)}_{0}} - 6\mathcal{L}(y) + 5\cdot\underbrace{\mathcal{L}\left(\int_0^x y(t)\,dt\right)}_{\frac{1}{p}\cdot\mathcal{L}(y)} = \frac{1}{p}$$

$$\mathcal{L}(y)\left(p - 6 + \frac{5}{p}\right) = \frac{1}{p}$$

$$\mathcal{L}(y) = \frac{1}{p^2 - 6p + 5} = \frac{1}{(p-5)(p-1)} = \frac{\overset{=\frac{1}{4}}{A}}{p-5} + \frac{\overset{=-\frac{1}{4}}{B}}{p-1}$$

$$1 = A(p-1) + B(p-5)$$

$$p=1:\; 1 = 0A - 4B \Rightarrow B = -\frac{1}{4}$$

$$p=5:\; 1 = 4A + 0B \Rightarrow A = \frac{1}{4}$$

$$\mathcal{L}^{-1}\left(\frac{\frac{1}{4}}{p-5} + \frac{\left(-\frac{1}{4}\right)}{p-1}\right)(t) = \underline{\frac{1}{4}e^{5t} - \frac{1}{4}e^{t}}$$

11. minitest RMF

Varianta B

13. 12. 2024

Pomocí Laplaceovy transformace řešte diferenciální rovnici

$$y'' - y' = 1$$

s počátečními podmínkami $y'(0) = 1$, $y(0) = 2$.

$$\mathcal{L}(y'' - y') = \mathcal{L}(1)$$

$$\mathcal{L}(y') = p \cdot \mathcal{L}(y) - y(0_+)$$

$$\mathcal{L}(y'') = p\,\mathcal{L}(y') - y'(0_+)$$

$$= p^2 \mathcal{L}(y) - p\,y(0_+) - y'(0_+)$$

$$p^2 \mathcal{L}(y) - p\underbrace{y(0_+)}_{2} - \underbrace{y'(0_+)}_{1} - \Big(p\mathcal{L}(y) - \underbrace{y(0_+)}_{2}\Big)$$

$$= \mathcal{L}(y)\left(p^2 - p\right) - 2p + 1 = \frac{1}{p}$$

$$\mathcal{L}(y) = \frac{\frac{1}{p} + 2p - 1}{p^2 - p} = \frac{2p^2 - p + 1}{p^3 - p^2} = \frac{2p^2 - p + 1}{p^2(p-1)}$$

$$\frac{2p^2 - p + 1}{p^2(p-1)} = \frac{A^{=-1}}{p^2} + \frac{B^{=0}}{p} + \frac{C^{=2}}{p-1}$$

$$2p^2 - p + 1 = A(p-1) + Bp(p-1) + Cp^2$$

$p=1$: $\quad 2 = 0A + 0B + C \Rightarrow \boxed{C = 2}$

$p=0$: $\quad 1 = -A + 0B + 0C \Rightarrow \boxed{A = -1}$

$$\underline{2p^2} - p + 1 = -p + 1 + \underline{Bp^2} - Bp + \underline{2p^2} \Rightarrow \boxed{B = 0}$$

$$y = \mathcal{L}^{-1}\left(-\frac{1}{p^2} + \frac{2}{p-1}\right) = \underline{\underline{-x + 2e^x}}$$