8. minitest RMF

Varianta A

22. 11. 2024

Vypočtěte Fourierovu transformaci funkce

$$f(x) = \frac{1}{2} e^{-|x|}$$

$$\mathcal{F}\left(\tfrac{1}{2}e^{-|x|}\right) = \int_{\mathbb{R}} \tfrac{1}{2} e^{-|x|} \cdot e^{ixt}\, dx = \frac{1}{2}\int_0^{\infty} e^{x(it-1)}\, dx +$$

$$+ \cdot \frac{1}{2}\int_{-\infty}^{0} e^{x(it+1)}\, dx = \frac{1}{2}\cdot\left(\left[\frac{e^{x(it-1)}}{it-1}\right]_0^{\infty} + \left[\frac{e^{x(it+1)}}{it+1}\right]_{-\infty}^{0}\right)$$

$$= \frac{1}{2}\left(\left(\lim_{x\to\infty} \frac{1}{it-1}\cdot e^{-x}\left(\cos(xt) + i\sin(xt)\right)\right) - \frac{1}{it-1}\right)$$

0 („0-omezená funkce")

$$+ \frac{1}{it+1} - \lim_{x\to-\infty}(\;)\Big)\Big) = \frac{1}{2}\left(-\frac{1}{it-1} + \frac{1}{it+1}\right)$$

0

$$= \frac{1}{2}\cdot\frac{-it-1+it-1}{(it)^2-1} = \frac{1}{2}\cdot\frac{(-2)}{-t^2-1} = \frac{1}{t^2+1}$$

8. minitest RMF

Varianta B

22. 11. 2024

Vypočtěte Fourierovu transformaci funkce

$$f(x) = \begin{cases} 1 & x \in [0,1) \\ -1 & x \in (-1,0) \\ 0 & |x| \geq 1 \end{cases}$$

$$\mathcal{F}(f(x))(t) = \int_{\mathbb{R}} f(x)\, e^{ixt}\, dx = \int_0^1 e^{ixt}\, dx - \int_{-1}^0 e^{ixt}\, dx =$$

$$= \left[\frac{e^{ixt}}{it} \right]_0^1 - \left[\frac{e^{ixt}}{it} \right]_{-1}^0 = \frac{1}{it}\left(e^{it} - 1\right)$$

$$- \left(\frac{1}{it} - \frac{e^{-it}}{it} \right) = -\frac{2}{it} + \frac{1}{it}\left(e^{it} + e^{-it}\right)$$

$\cos t + i\sin t + \cos(-t) + i\sin(-t)$

$$= \frac{1}{it}\cdot\left(-2 + 2\cos t\right)$$