Varianta A

1a) $\int_1^2 \frac{x^3}{x^4-1}\,dx = \frac{1}{4}\int_0^{15} \frac{1}{t}\,dt = \frac{1}{4}\left[\ln|t|\right]_0^{15} =$

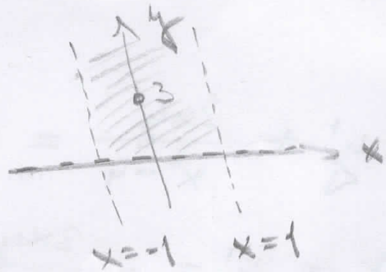
$\left| \begin{array}{l} x^4-1 = t \\ 4x^3\,dx = dt \\ x^3\,dx = \frac{1}{4}\,dt \end{array} \right|$

$= \frac{1}{4}\left(\ln 15 - \underbrace{\lim_{t\to 0_+} \ln|t|}_{=-\infty}\right) = \underline{\underline{+\infty}}$ => DIVERGUJE

1b) $\int_0^1 \frac{1}{\sqrt{x\sqrt{x}}}\,dx = \int_0^1 x^{-\frac{3}{4}}\,dx = \left[4x^{\frac{1}{4}}\right]_0^1 = \underline{\underline{4}}$ => KONVERGUJE

$\left(x^1 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(x^{\frac{3}{2}}\right)^{\frac{1}{2}} = x^{\frac{3}{4}}$

2) $y' = \frac{4xy}{x^2-1} \quad \Big| : y \quad \Big| \cdot dx$



$\int \frac{1}{y}\,dy = \int \frac{4x}{x^2-1}\,dx$

$\ln|y| = 2\ln|x^2-1| + c$

$x \neq \pm 1$

$\ulcorner \int \frac{4x}{x^2-1}\,dx = 2\int \frac{1}{t}\,dt = 2\ln|x^2-1|$

$\left| \begin{array}{l} x^2-1 = t \\ 2x\,dx = dt \end{array} \right| \lrcorner$

a) $\underline{y(0)=3}$: $\ln 3 = c$

$\ln y = \underbrace{\ln(x^2-1)^2 + \ln 3}_{\ln\left(3\cdot(x^2-1)^2\right)}$

$\boxed{y = 3\cdot(x^2-1)^2}, \; x \in (-1,1)$

b) $\underline{y(0)=0}$: $\boxed{y=0}$

Varianta B

1a) $\int_0^{\infty} \frac{e^x}{e^{2x}+1}\,dx = \int_0^{\infty} \frac{e^x}{(e^x)^2+1}\,dx = \int_1^{\infty} \frac{1}{t^2+1}\,dt = [\operatorname{arctg} t]_1^{\infty} =$

$\left| \begin{array}{l} e^x = t \\ e^x dx = dt \end{array} \right|$

$= \lim_{t\to\infty} \operatorname{arctg} t - \operatorname{arctg} 1 = \frac{\pi}{2} - \frac{\pi}{4} = \underline{\underline{\frac{\pi}{4}}}$ => KONVERGUJE

1b) $\int_0^1 \frac{1+\frac{1}{x}}{1-\frac{1}{x}}\,dx = \int_0^1 \frac{\frac{x+1}{x}}{\frac{x-1}{x}}\,dx = \int_0^1 \frac{x+1}{x-1}\,dx = \int_0^1 \left(1+\frac{2}{x-1}\right)dx$

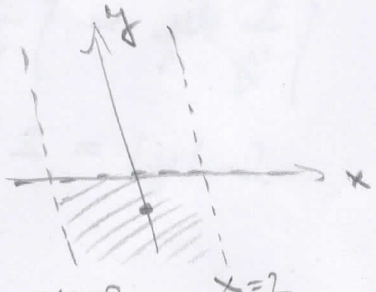
$(x+1):(x-1) = 1 + \frac{2}{x-1}$
$\underline{-(x-1)}$
2

$= [x + 2\ln|x-1|]_0^1$
$= \lim_{x\to 1_-}(x+2\ln|x-1|) - 2\ln 1 = 1 + 2\ln 0_+ = \underline{\underline{-\infty}}$

=> DIVERGUJE

2) $y' + \frac{2xy}{x^2-4} = 0$ $\qquad x \neq \pm 2$

$\frac{dy}{dx} = y' = -\frac{2xy}{x^2-4}$ $\quad |\cdot dx$ $\quad |:y$



$\int \frac{1}{y}\,dy = \int \left(-\frac{2x}{x^2-4}\right)dx$

$\ln|y| = -\ln|x^2-4| + c$

$(-2,2)\times(-\infty,0)$

a) $\underline{y(0)=-1}$: $\ln 1 = -\ln 4 + c \implies c = \ln 4$

$\ln|y| = \ln\frac{4}{|x^2-4|}$

$-y = -\frac{4}{x^2-4} \implies \boxed{y = \frac{4}{x^2-4}}$ $\quad x\in(-2,2)$

b) $\underline{y(0)=0}$: $\boxed{y=0}$