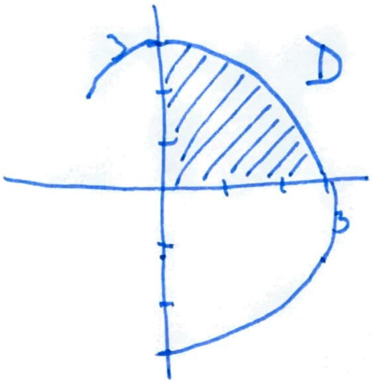
1) $\displaystyle\iint_D \ln(1+x^2+y^2)\,dx\,dy$

$D = \{(x,y) \in \mathbb{R}^2;\ x^2+y^2 \le 9 \wedge x \ge 0 \wedge y \ge 0\}$



substituce do polárních souřadnic

$x = r\cos\varphi$
$y = r\sin\varphi$ $\qquad J(r,\varphi) = r$

$H = \{(r,\varphi) \in \mathbb{R}^2;\ r \in (0,3\rangle;\ \varphi \in \langle 0, \frac{\pi}{2}\rangle\}$

$H \longrightarrow D \setminus \{(0,0)\}$

$$I = \iint_H \ln(1+r^2)\, r\, dr\, d\varphi = \int_0^3\int_0^{\frac{\pi}{2}} \ln(1+r^2)\, r\, d\varphi\, dr$$

$$= \left(\int_0^{\frac{\pi}{2}} 1\, d\varphi\right)\cdot\left(\int_0^3 \ln(1+r^2)\, r\, dr\right) = \frac{\pi}{2}\cdot\int_0^3 \ln(1+r^2)\, r\, dr = *$$

$$\int \ln(1+r^2)\, r\, dr = \left| \begin{matrix} u = 1+r^2 \\ du = 2r\,dr \end{matrix} \right| = \frac{1}{2}\int \ln u\, du =$$

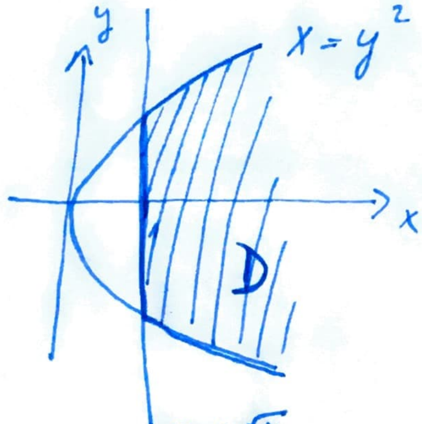
$$\left| \begin{matrix} \ln u & \to & \frac{1}{u} \\ 1 & \to & u \end{matrix} \right| = \frac{1}{2} u \ln u - \frac{1}{2}\int 1\, du = \frac{1}{2}(1+r^2)\ln(1+r^2) - \frac{1}{2}(1+r^2)$$

$$* = \frac{\pi}{2}\left[\frac{1}{2}(1+r^2)\left(\ln(1+r^2) - 1\right)\right]_0^3 = \frac{\pi}{4}\left(10\ln 10 - 10 - (-1)\right)$$

$$= \frac{\pi}{4}\left(10\ln 10 - 9\right)$$

(2) $$\iint_D \frac{11(\sqrt{x}+y)^{10}}{x^{10}}\,dx\,dy$$

$$D = \{(x,y) \in \mathbb{R}^2;\ y^2 - x \leq 0 \land x \geq 1\}$$



$$D = \{(x,y) \in \mathbb{R}^2;\ x \in \langle 1, \infty);\ -\sqrt{x} \leq y \leq \sqrt{x}\}$$

$$I = \int_1^\infty \left(\int_{-\sqrt{x}}^{\sqrt{x}} \frac{11(\sqrt{x}+y)^{10}}{x^{10}}\,dy \right) dx = \int_1^\infty \left[\frac{(\sqrt{x}+y)^{11}}{x^{10}} \right]_{-\sqrt{x}}^{\sqrt{x}} dx$$

$$= \int_1^\infty \frac{(2\sqrt{x})^{11}}{x^{10}}\,dx = 2^{11} \int_1^\infty x^{\frac{11}{2}-10}\,dx = 2^{11} \left[\frac{x^{-\frac{7}{2}}}{-\frac{7}{2}} \right]_1^\infty =$$

$$= -\frac{2^{12}}{7} \left(\lim_{x \to \infty} x^{-\frac{7}{2}} - 1 \right) = \underline{\underline{\frac{2^{12}}{7}}}$$

③ $x' = -x - y = f(t, x, y)$ $\quad x(0) = 1$

$y' = 5xy + 10t = g(t, x, y)$ $\quad y(0) = 1$

$h = 0{,}1$ $\quad t_{i+1} = t_i + h$ $\quad t_0 = 0$

$x_{i+1} = x_i + h f(t_i, x_i, y_i)$ $\quad x_0 = 1$

$y_{i+1} = y_i + h \cdot g(t_i, x_i, y_i)$ $\quad y_0 = 1$

i	t_i	x_i	y_i	$h \cdot f(t_i, x_i, y_i)$	$h \cdot g(t_i, x_i, y_i)$
0	0	1	1	$0{,}1 \cdot (-1-1) = -0{,}2$	$0{,}1 \cdot (5 \cdot 1 \cdot 1 + 0) = 0{,}5$
1	0,1	0,8	1,5	$0{,}1(-0{,}8-1{,}5) = -0{,}23$	$0{,}1 \cdot (5 \cdot 0{,}8 \cdot 1{,}5 + 1) = 0{,}7$
2	0,2	0,57	2,2		

$\vec{z}(0{,}2) \doteq (0{,}57 \,;\, 2{,}2)$