

MGI - 10. minuter, 10.12.2025

$$A = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$$

a) Učíte vlastní čísla a vlastní vektory.

$$\det(A - \lambda I) = \begin{vmatrix} -\frac{4}{5} - \lambda & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} - \lambda \end{vmatrix} = \left(-\frac{4}{5} - \lambda\right)\left(\frac{4}{5} - \lambda\right) - \frac{3}{5} \cdot \frac{3}{5}$$

$$= -\frac{16}{25} - \frac{4}{5}\lambda + \frac{3}{5}\lambda + \lambda^2 - \frac{9}{25} = \lambda^2 - 1$$

$$\det(A - \lambda I) = 0 \iff \begin{aligned} \lambda^2 - 1 &= 0 \\ (\lambda - 1)(\lambda + 1) &= 0 \\ \lambda &= 1 \vee \lambda = -1 \end{aligned}$$

$\lambda = 1$: Hledáme $\vec{x} \neq \vec{0}$ tak, že $(A - \lambda I)\vec{x} = \vec{0}$

$$\begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{1}{5} \end{pmatrix} \begin{array}{l} \vec{x} \\ \vec{y} \end{array} \left| \begin{array}{l} 0 \\ 0 \end{array} \right.$$

$\lambda = -1$: $(A - (-1)I)\vec{x} = \vec{0}$

$$\begin{pmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{9}{5} \end{pmatrix} \begin{array}{l} \vec{x} \\ \vec{y} \end{array} \left| \begin{array}{l} 0 \\ 0 \end{array} \right.$$

$$\vec{x} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

b) Ověřme, že matice A je matice shodnosti.

$$A^T A = \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix} \begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Ans.

Osová souměrnost podle osy $y = 3x$.

