11. minitest MG1, 17.12.2025

Najděte singulární rozklad matice $A = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$.

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$$A^T A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix}$$

Hledáme vlastní čísla a vlastní vektory matice $A^T A$.

$$\begin{vmatrix} 2-\lambda & 4 \\ 4 & 8-\lambda \end{vmatrix} = (2-\lambda)(8-\lambda) - 16 = 16 - 8\lambda - 2\lambda + \lambda^2 - 16$$
$$= \lambda^2 - 10\lambda$$
$$= \lambda(\lambda - 10)$$

I. $\lambda_1 = 10$ ; $\sigma_1 = \sqrt{10}$ $\qquad (A - 10I)\vec{x} = \vec{0}$ $\qquad \begin{pmatrix} -8 & 4 & | & 0 \\ 4 & -2 & | & 0 \end{pmatrix}$ $\qquad \vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\qquad |\vec{x}| = \sqrt{5}$

II. $\lambda_2 = 0$ ; $\sigma_2 = 0$ $\qquad (A - 0I)\vec{x} = \vec{0}$ $\qquad \begin{pmatrix} 2 & 4 & | & 0 \\ 4 & 8 & | & 0 \end{pmatrix}$ $\qquad \vec{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $\qquad |\vec{x}| = \sqrt{5}$

$$A = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}}_{U} \cdot \underbrace{\begin{pmatrix} \sqrt{10} & 0 \\ 0 & 0 \end{pmatrix}}_{\Sigma} \cdot \underbrace{\begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}^T}_{V^*}$$

2. sloupec je vektor $\vec{u}_2$ kolmý k $\vec{u}_1$ takový, že $|\vec{u}_2| = 1$

1. sloupec je dán vztahem

$$\vec{u}_1 = \sigma_1^{-1} \cdot A \cdot \vec{v}_1 = \frac{1}{\sqrt{10}} \cdot \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} = \frac{1}{\sqrt{10}} \cdot \begin{pmatrix} \frac{5}{\sqrt{5}} \\ \frac{5}{\sqrt{5}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$