

9. minutes MGI, 3.12.

$$z = \frac{1}{\frac{1}{2+i} + i} + \frac{1}{\frac{1}{1-2i} + 1}$$

$$a) \quad \frac{1}{2+i} \cdot \frac{2-i}{2-i} = \frac{2-i}{4-i^2} = \frac{2-i}{4-(-1)} = \frac{2-i}{5}$$

$$\frac{1}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{1+2i}{1-4i^2} = \frac{1+2i}{1+4} = \frac{1+2i}{5}$$

$$z = \frac{1}{\frac{2-i}{5} + i} + \frac{1}{\frac{1+2i}{5} + 1} = \frac{5}{2-i+5i} + \frac{5}{1+2i+5} =$$

$$= \frac{5}{4i+2} \cdot \frac{4i-2}{4i-2} + \frac{5}{6+2i} \cdot \frac{6-2i}{6-2i} =$$

$$= \frac{20i-10}{16i^2-4} + \frac{30-10i}{36-4i^2} = \frac{20i-10}{(-20)} + \frac{30-10i}{40} =$$

$$= -i + \frac{1}{2} + \frac{3}{4} - \frac{1}{4}i = \underline{\underline{\frac{5}{4} - \frac{5}{4}i}}$$

b)

$$|z| = \sqrt{\left(\frac{5}{4}\right)^2 + \left(\frac{5}{4}\right)^2} = \sqrt{\frac{50}{16}} = \frac{5\sqrt{2}}{4}$$

$$z = \frac{5\sqrt{2}}{4} \cdot \left(\cos\left(\frac{7}{4}\pi\right) + i \sin\left(\frac{7}{4}\pi\right) \right) = \frac{5\sqrt{2}}{4}$$

c)

$$z^4 = \left(\frac{5\sqrt{2}}{4}\right)^4 \cdot \left(\cos\left(4 \cdot \frac{7}{4}\pi\right) + i \sin\left(4 \cdot \frac{7}{4}\pi\right) \right)$$

$$= \left(\frac{25}{8}\right)^2 \cdot \left(\underbrace{\cos 7\pi = \cos \pi = -1}_{\cos 7\pi = \cos \pi = -1} + \underbrace{i \sin 7\pi = 0}_{\sin 7\pi = 0} \right) = -\frac{625}{64}$$