# 2. minitest

Matematická analýza 1, ZS 2025/26

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V závislosti na parametru $\alpha \in \mathbb{R}^+$ vypočtěte limitu posloupnosti

$$\lim_{n\to\infty}\left(\frac{\alpha n^2-3n+1}{2n^2+3n+1}\right)^n$$

$$\lim_{n\to\infty}\frac{\alpha n^2-3n+1}{2n^2+3n+1}=\lim_{n\to\infty}\frac{\cancel{n^2}\left(\alpha-\frac{3}{n}+\frac{1}{n^2}\right)}{\cancel{n^2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)}=\frac{\alpha-0+0}{2+0+0}=\frac{\alpha}{2}$$

I. $\alpha\in\mathbb{R}^+\setminus\{2\}$: $\lim_{n\to\infty}\left(\frac{\alpha n^2-3n+1}{2n^2+3n+1}\right)^n=\left(\frac{\alpha}{2}\right)^\infty=\begin{cases}\infty, & \alpha>2\\ 0, & \alpha\in(0,2)\end{cases}$

II. $\alpha=2$: $\lim_{n\to\infty}\left(\frac{2n^2-3n+1\overset{+3n-3n}{}}{2n^2+3n+1}\right)^n=\lim_{n\to\infty}\left(1-\frac{6n}{2n^2+3n+1}\right)^n=$

$$=\lim_{n\to\infty}\left(\left(1+\frac{(-1)}{\frac{2n^2+3n+1}{6n}}\right)^{\frac{2n^2+3n+1}{6n}}\right)^{\frac{6n}{2n^2+3n+1}\cdot n}$$

$$=\Big(\underbrace{\lim_{n\to\infty}(\quad)}_{e^{-1}}\Big)^{\lim\frac{6n^2}{2n^2+3n+1}}=\left(e^{-1}\right)^3=e^{-3}$$

$$\lim\frac{6n^2}{2n^2+3n+1}=\lim\frac{6}{2+\frac{3}{n}+\frac{1}{n^2}}=\frac{6}{2+0+0}=3$$