

3. minitest

Matematická analýza 1, ZS 2025/26

27. 11. 2025

V závislosti na parametru $\alpha \in \mathbb{R}$ vypočítejte limitu posloupnosti

$$\lim_{n \rightarrow \infty} n^\alpha (\sqrt{n^5 + 3n} - \sqrt{n^5 - 3n})$$

$$\lim_{n \rightarrow \infty} n^\alpha (\sqrt{n^5 + 3n} - \sqrt{n^5 - 3n}) = \frac{\sqrt{n^5 + 3n} + \sqrt{n^5 - 3n}}{\sqrt{n^5 + 3n} + \sqrt{n^5 - 3n}} =$$

$$= \lim_{n \rightarrow \infty} n^\alpha \cdot \frac{(n^5 + 3n) - (n^5 - 3n)}{\sqrt{n^5 + 3n} + \sqrt{n^5 - 3n}} =$$

$$= \lim_{n \rightarrow \infty} n^\alpha \cdot \frac{6n}{n^{\frac{5}{2}} \left(\sqrt{1 + \frac{3}{n^4}} + \sqrt{1 - \frac{3}{n^4}} \right)} =$$

$$\stackrel{\text{VOAL}}{=} \left(\lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{3}{n^4}} + \sqrt{1 - \frac{3}{n^4}}} \right) \cdot \left(\lim_{n \rightarrow \infty} n^{\alpha - \frac{3}{2}} \right) = \begin{cases} +\infty, & \alpha > \frac{3}{2} \\ 0, & \alpha < \frac{3}{2} \\ 3, & \alpha = \frac{3}{2} \end{cases}$$

$$= \frac{6}{\sqrt{1+0} + \sqrt{1-0}} = \frac{6}{2} = 3$$