4. minitest

Matematická analýza 1, ZS 2025/26

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Vypočtěte limitu funkce

$$\lim_{x\to 0} \frac{1-\sqrt{\cos x}}{\ln(\cos 2x)}$$

$$\lim_{x\to 0} \frac{1-\sqrt{\cos x}}{\ln(\cos 2x)} \cdot \frac{1+\sqrt{\cos x}}{1+\sqrt{\cos x}} \overset{\text{VOAL}}{=} \underbrace{\lim_{x\to 0} \frac{1}{1+\sqrt{\cos x}}}_{\frac{1}{2}} \cdot \lim_{x\to 0} \frac{1-\cos x}{\ln(\cos 2x)}$$

$$\overset{\text{VOAL}}{=} \frac{1}{2} \cdot \underbrace{\lim_{x\to 0} \frac{1-\cos x}{x^2}}_{\frac{1}{2}} \cdot \lim_{x\to 0} \frac{x^2}{\ln(\cos 2x)} \cdot \frac{\cos(2x)-1}{\cos(2x)-1} =$$

$$\overset{\text{VOAL}}{=} \frac{1}{4} \cdot \left(\lim_{x\to 0} \frac{x^2}{\cos(2x)-1}\right) \cdot \frac{1}{\underbrace{\lim_{x\to 0} \frac{\ln(\cos 2x)}{\cos(2x)-1}}_{=1 \text{ VOLSF}}} =$$

$$= \frac{1}{4} \cdot (-1) \cdot \frac{1}{4 \cdot \underbrace{\lim_{x\to 0} \frac{1-\cos(2x)}{4x^2}}_{=\frac{1}{2} \text{ VOLSF}}}$$

$g(x) = \cos 2x \xrightarrow[x\to 0]{} 1$

$f(y) = \frac{\ln y}{y-1} \xrightarrow[y\to 1]{} 1$

$g(x) = 2x \xrightarrow[x\to 0]{} 0$

$f(y) = \frac{1-\cos y}{y^2} \xrightarrow[y\to 0]{} \frac{1}{2}$

$$= -\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{\frac{1}{2}} = -\frac{1}{8}$$