5. minitest MAN1, 15.12.2025

$$\lim_{x\to 0}\left(\frac{1+\sin 2x}{1+\sin x}\right)^{\cot g x} = \lim_{x\to 0} e^{\cot g x\cdot \ln\left(\frac{1+\sin 2x}{1+\sin x}\right)}$$

$$\overset{\text{VOLSF}}{=} e^{\lim_{x\to 0}\cot g x\cdot \ln\left(\frac{1+\sin 2x}{1+\sin x}\right)} \overset{*}{=} e^{1} = \underline{\underline{e}}$$

$$(*)\quad \lim_{x\to 0}\cot g x\cdot \ln\left(\frac{1+\sin 2x}{1+\sin x}\right)\cdot\frac{\frac{1+\sin 2x}{1+\sin x}-1}{\frac{1+\sin 2x}{1+\sin x}-1} =$$

$$\overset{\text{VOAL}}{=} \underbrace{\left(\lim_{x\to 0}\frac{\ln\left(\frac{1+\sin 2x}{1+\sin x}\right)}{\frac{1+\sin 2x}{1+\sin x}-1}\right)}_{\overset{=1}{\text{VOLSF}}}\cdot\left(\lim_{x\to 0}\frac{\cos x}{\sin x}\cdot\frac{1+\sin 2x-(1+\sin x)}{1+\sin x}\right)$$

$$g(x)=\frac{1+\sin 2x}{1+\sin x}\xrightarrow[x\to 0]{} 1$$

$$f(y)=\frac{\ln y}{y-1}\xrightarrow[y\to 1]{} 1$$

$$\overset{\text{VOAL}}{=}\underbrace{\left(\lim_{x\to 0}\cos x\right)}_{1}\cdot\underbrace{\left(\lim_{x\to 0}\frac{1}{1+\sin x}\right)}_{1}\cdot\left(\lim_{x\to 0}\frac{\sin 2x-\sin x}{\sin x}\right) = 1$$

$$\lim_{x\to 0}\left(\frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}}-1\right)$$

$$2\cos 0-1=1$$

nebo:
$$\lim_{x\to 0}\left(\frac{1+\overset{\sim 2x}{\sin 2x}}{1+\underset{\sim x}{\sin x}}\right)^{\overset{\sim\frac{1}{x}}{\frac{\cos x}{\sin x}}} = \lim_{x\to 0}\left(\frac{1+2x}{1+x}\right)^{\frac{1}{x}} =$$

$$= \lim_{x\to 0}\left(1+\frac{x}{1+x}\right)^{\frac{1}{x}} = \lim_{x\to 0}\underbrace{\left(\left(1+\frac{1}{\frac{1+x}{x}}\right)^{\frac{1+x}{x}}\right)}_{\to e}^{\overset{\xrightarrow[x\to 0]{}1}{\frac{1}{1+x}}} = \underline{\underline{e^{1}}}$$