6. minitest

Matematika pro geoinformatiky, ZS 2025/26

12. 11. 2025

Je dána funkce

$$f(x,y) = \sqrt[3]{x^2 - y^3}$$

a) Určete její gradient v bodě $[3,1]$.

b) Sestavte Taylorův polynom 1. stupně v tomto bodě (rovnici tečné roviny).

c) Pomocí něj vypočtěte přibližnou hodnotu

$$\sqrt[3]{3,02^2 - 0,92^3}$$

a) $$\frac{\partial f}{\partial x} = 2x \cdot \frac{1}{3} \cdot (x^2 - y^3)^{-\frac{2}{3}} \Big|_{[3,1]} = 2 \cdot 3 \cdot \frac{1}{3} \cdot \frac{1}{\left(\sqrt[3]{3^2 - 1^3}\right)^2} = \frac{2}{\left(\sqrt[3]{8}\right)^2} = \underline{\underline{\frac{1}{2}}}$$

$$\frac{\partial f}{\partial y} = (-3y^2) \cdot \frac{1}{3} \cdot (x^2 - y^3)^{-\frac{2}{3}} \Big|_{[3,1]} = (-3) \cdot \frac{1}{3} \cdot \frac{1}{\left(\sqrt[3]{8}\right)^2} = \underline{\underline{-\frac{1}{4}}}$$

$$\boxed{\nabla f(3,1) = \left(1, -\frac{1}{2}\right)}$$

b) $$T(x,y) = \underbrace{f(3,1)}_{\sqrt[3]{9-1} = \sqrt[3]{8} = 2} + \underbrace{\frac{\partial f}{\partial x}(3,1)}_{\frac{1}{2}} \cdot (x-3) + \underbrace{\frac{\partial f}{\partial y}(3,1)}_{\left(-\frac{1}{4}\right)} \cdot (y-1)$$

$$\boxed{T(x,y) = 2 + \frac{1}{2}(x-3) - \frac{1}{4}(y-1)}$$

c) $$f(3{,}02; 0{,}92) \doteq T(3{,}02; 0{,}92)$$

$$\sqrt[3]{3{,}02^2 - 0{,}92^3} \doteq 2 + \frac{1}{2}(3{,}02 - 3) - \frac{1}{4}(0{,}92 - 1)$$

$$= 2 + 0{,}01 - \frac{1}{4} \cdot (-0{,}08)$$

$$= 2{,}01 + 0{,}02 = \underline{\underline{2{,}03}}$$