

7. mini test

Najděte lokální extrémum a sedlové body funkce

$$f(x, y) = x^3 + \frac{1}{2}y^2 + 15x^2 - 6xy + 12x - 4y$$

$$\frac{\partial f}{\partial x} = 3x^2 + 30x - 6y + 12 = 0 \quad | :3 \quad \text{dosadíme}$$

$$\frac{\partial f}{\partial y} = 2y - 6x - 4 = 0 \quad \Leftrightarrow \boxed{y = 3x + 2}$$

$$x^2 + 10x - 2 \cdot (3x + 2) + 4 = 0$$

$$x^2 + 4x = 0$$

$$x(x + 4) = 0$$

$$x = 0 \quad \vee \quad x = -4$$

$$y = 2$$

$$y = -10$$

$$\text{Hessova matice } H(x, y) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} = \begin{pmatrix} 6x + 30 & -6 \\ -6 & 2 \end{pmatrix}$$

$$H(0, 2) = \begin{pmatrix} 30 & -6 \\ -6 & 2 \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial x^2}(0, 2) = 30 > 0$$

$$\wedge \det H(0, 2) = 30 \cdot 2 - (-6) \cdot (-6) > 0$$

\Rightarrow POZITIVNĚ DEFINITNÍ MATICE

$\Rightarrow [0, 2]$ JE LOK. MINIMUM

$$H(-4, -10) = \begin{pmatrix} 6 & -6 \\ -6 & 2 \end{pmatrix}$$

$$\det H = 6 \cdot 2 - (-6) \cdot (-6) = 12 - 36 < 0$$

\Rightarrow INDEFINITNÍ MATICE

$\Rightarrow [-4, -10]$ JE SEDLOVÝ BOD