

11. minitest  
 Matematika M2 (b), LS 2025/26  
 4. 5. 2026

Vypočtěte směrovou derivaci funkce

$$f(x, y) = \sqrt[3]{x^3 - y^2}$$

a) v bodě  $A = [1, 3]$  ve směru vektoru  $\vec{a} = (3, 4)^T$ .

b) v bodě  $B = [0, 0]$  ve směru vektoru  $\vec{b} = (1, 0)^T$ .

$$a) \quad |\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\frac{\partial f}{\partial x}(x, y) = \frac{1}{3} \cdot (x^3 - y^2)^{-\frac{2}{3}} \cdot 3x^2 = \frac{x^2}{3\sqrt[3]{(x^3 - y^2)^2}}$$

$$\frac{\partial f}{\partial y}(x, y) = -\frac{1}{3} \cdot (x^3 - y^2)^{-\frac{2}{3}} \cdot 2y = -\frac{2y}{3\sqrt[3]{(x^3 - y^2)^2}}$$

$$\nabla f(1, 3) = \left( \frac{1}{3\sqrt[3]{64}}, -\frac{6}{3 \cdot 3\sqrt[3]{64}} \right)^T = \left( \frac{1}{4}, -\frac{1}{2} \right)^T$$

$$\begin{aligned} \frac{\partial f}{\partial \vec{a}}(A) &= \nabla f(A) \cdot \frac{\vec{a}}{|\vec{a}|} = \left( \frac{1}{4}, -\frac{1}{2} \right) \cdot \left( \frac{3}{5}, \frac{4}{5} \right) = \\ &= \frac{1}{4} \cdot \frac{3}{5} + \left( -\frac{1}{2} \cdot \frac{4}{5} \right) = \frac{3}{20} - \frac{2}{5} = \\ &= \frac{3 - 8}{20} = \underline{\underline{-\frac{1}{4}}} \end{aligned}$$

$$b) \quad \text{Z definice: } \frac{\partial f}{\partial \vec{b}}(B) = \lim_{t \rightarrow 0} \frac{f(B + t\vec{b}) - f(B)}{t} =$$

$$= \lim_{t \rightarrow 0} \frac{f(t, 0)}{t} = \lim_{t \rightarrow 0} \frac{\sqrt[3]{t^3 - 0^2}}{t} = \lim_{t \rightarrow 0} \frac{t}{t} = \underline{\underline{1}}$$