12. minitest

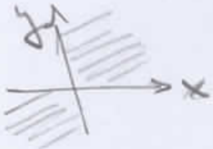
Matematika M2 (b), LS 2025/26

11. 5. 2026

Najděte lokální extrémy a sedlové body funkce

$$f(x,y) = 2\ln(xy) - x^2 - y$$

Definiční obor: $xy > 0 \iff (x>0 \wedge y>0) \vee (x<0 \wedge y<0)$



$D_f = \{(x,y) \in \mathbb{R}^2;\ xy > 0\}$

$\frac{\partial f}{\partial x}(x,y) = 2 \cdot \frac{1}{xy} \cdot y - 2x = \frac{2}{x} - 2x = 0 \quad | :2,\ | \cdot x$

$\frac{\partial f}{\partial y}(x,y) = 2 \cdot \frac{1}{xy} \cdot x - 1 = \frac{2}{y} - 1 = 0 \quad | \cdot y$

$1 - x^2 = 0 \iff x = \pm 1$

$2 - y = 0 \iff y = 2$

stac. bod: $[1,2] \in D_f$

$[-1,2] \notin D_f$

Matice 2. parciálních derivací

$$H(x,y) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} = \begin{pmatrix} -\frac{2}{x^2} - 2 & 0 \\ 0 & -\frac{2}{y^2} \end{pmatrix}$$

$H(1,2) = \begin{pmatrix} -4 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$ negativně definitní matice

neboť $\det H = 2 > 0$

a $\frac{\partial^2 f}{\partial x^2}(1,2) < 0$

$\implies [1,2]$ je lok. maximum