4. minitest MA1

Varianta A

15. 11. 2024

Vypočtěte limitu posloupnosti.

$$\lim_{n\to\infty}\left(\frac{3n^2+n+1}{3n^2-n+1}\right)^n$$

$$= \lim_{n\to\infty}\left(\frac{3n^2-n+1+2n}{3n^2-n+1}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{\frac{3n^2-n+1}{2n}}\right)^n =$$

$$= \lim_{n\to\infty}\left(\left(1+\frac{1}{\frac{3}{2}n-\frac{1}{2}+\frac{1}{2n}}\right)^{\frac{3}{2}n-\frac{1}{2}+\frac{1}{2n}}\right)^{\frac{n}{\frac{3}{2}n-\frac{1}{2}+\frac{1}{2n}}} =$$

$$= e^{\frac{2}{3}} = \sqrt[3]{e^2}$$

$$\lim_{n\to\infty}\frac{n}{\frac{3}{2}n-\frac{1}{2}+\frac{1}{2n}} = \lim_{n\to\infty}\frac{n}{n\left(\frac{3}{2}-\frac{1}{2n}+\frac{1}{2n^2}\right)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

4. minitest MA1

Varianta B

15. 11. 2024

Vypočtěte limitu posloupnosti.

$$\lim_{n\to\infty}\left(\frac{4n^2+n-1}{4n^2-n+1}\right)^n$$

$$= \lim_{n\to\infty}\left(\frac{4n^2-n+1+2n-2}{4n^2-n+1}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{\frac{4n^2-n+1}{2n-2}}\right)^n =$$

$$= \lim_{n\to\infty}\left(\left(1+\frac{1}{\frac{4n^2-n+1}{2n-2}}\right)^{\frac{4n^2-n+1}{2n-2}}\right)^{\frac{2n-2}{4n^2-n+1}\cdot n} =$$

$$= e^{\frac{1}{2}} = \sqrt{e}$$

$$\lim_{n\to\infty}\frac{2n^2-2n}{4n^2-n+1} = \lim_{n\to\infty}\frac{n^2\left(2-\frac{2}{n}\right)}{n^2\left(4-\frac{1}{n}+\frac{1}{n^2}\right)} = \frac{2-0}{4-0+0} = \frac{1}{2}$$