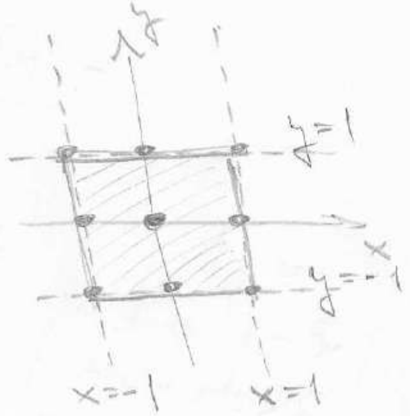
(1.) $f(x,y) = 3x^2 - y^3$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 6x = 0 \\ \frac{\partial f}{\partial y} &= -3y^2 = 0 \end{aligned}\right\}\ \text{stac. bod: } [0,0]$$



$\underline{x=\pm 1}$: $g(y) := f(\pm 1, y) = 3 - y^3$

$g'(y) = -3y^2 = 0 \Leftrightarrow y = 0$

$\underline{y=\pm 1}$: $g(x) := f(x, \pm 1) = 3x^2 \mp 1$

$g'(x) = 6x = 0 \Leftrightarrow x = 0$

podezřelé body: $[0,0]$, $[\pm 1,0]$, $[0,\pm 1]$, $[1,\pm 1]$, $[-1,\pm 1]$

$f(0,0) = 0$

$f(\pm 1,0) = 3$

$f(0,1) = -1$ minimum

$f(0,-1) = 1$

$f(\pm 1,1) = 2$

$f(\pm 1,-1) = 4$ maxima

(2a) $$\int_0^1 \frac{(x-1)^2}{\sqrt{x}}\,dx = \int_0^1 \frac{x^2-2x+1}{\sqrt{x}}\,dx = \int_0^1 \left(\frac{x^2}{\sqrt{x}} - 2\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right)$$

$$= \int_0 \left(x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx = \left[\frac{2}{5}x^{\frac{5}{2}} - 2\cdot\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\right]_0^1$$

$$= \frac{2}{5} - \frac{4}{3} + 2 = \frac{6-20+30}{15} = \underline{\underline{\frac{16}{15}}}$$

(2b) $\int \frac{x^3}{x^2-1}\,dx = \int \left(x + \frac{x}{x^2-1}\right)dx = \frac{x^2}{2} + \int \frac{x}{x^2-1}\,dx =$

$$x^3 : (x^2-1) = x + \frac{x}{x^2-1}$$
$$\underline{-(x^3 - x)}$$
$$\cancel{x}$$

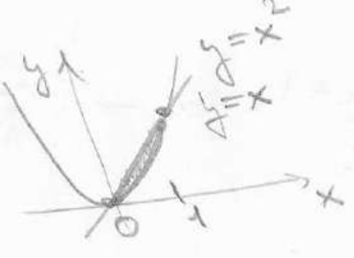
$$= \frac{x^2}{2} + \frac{1}{2}\int \frac{1}{t}\,dt = \underline{\underline{\frac{x^2}{2} + \frac{1}{2}\ln|x^2-1| + c}}$$

$$\left| \begin{array}{l} x^2-1 = t \\ 2x\,dx = dt \\ x\,dx = \frac{1}{2}dt \end{array} \right|$$

(2c) $\int x e^{2x}\,dx \overset{P.P.}{=} x \cdot \frac{1}{2}e^{2x} - \underbrace{\int \frac{1}{2}e^{2x}\,dx}_{= \frac{1}{4}e^{2x}} = \underline{\underline{e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right) + c}}$

$$\left| \begin{array}{ll} u = x & v' = e^{2x} \\ u' = 1 & v = \frac{1}{2}e^{2x} \end{array} \right|$$

(3.) $\iint_M \sqrt{xy}\,dy\,dx = \int_0^1 \int_{x^2}^{x} \sqrt{x}\cdot\sqrt{y}\,dy\,dx =$



$$= \int_0^1 \sqrt{x}\cdot\left[\frac{2}{3}y^{\frac{3}{2}}\right]_{x^2}^{x} dx =$$

$$= \int_0^1 x^{\frac{1}{2}}\cdot\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{3}x^3\right)dx =$$

$$= \frac{2}{3}\cdot\int_0^1 \left(x^2 - x^{\frac{7}{2}}\right)dx = \frac{2}{3}\cdot\left[\frac{x^3}{3} - \frac{2}{9}x^{\frac{9}{2}}\right]_0^1 =$$

$$= \frac{2}{3}\cdot\underbrace{\left(\frac{1}{3} - \frac{2}{9}\right)}_{\frac{1}{9}} = \frac{2}{3}\cdot\frac{1}{9} = \underline{\underline{\frac{2}{27}}}$$

(4.) $\sum_{n=1}^{\infty} \underbrace{\frac{2^{n+1}+2^{n-2}}{2^{n-1}+2^{n+2}}}_{a_n}$

Nutná podmínka konvergence řady: $\lim_{n\to\infty} a_n \stackrel{?}{=} 0$

$$\lim a_n = \lim_{n\to\infty} \frac{\cancel{2^n}\left(2+\frac{1}{4}\right)}{\cancel{2^n}\left(\frac{1}{2}+4\right)} = \frac{\frac{9}{4}}{\frac{9}{2}} = \frac{1}{2} \neq 0$$

$\Rightarrow$ Řada diverguje.

(5.) $$\sum_{n=0}^{\infty} \frac{2^{n+2}\cdot 3^{n+1}}{7^n+7^{n+1}} = \sum_{n=0}^{\infty} \frac{2^n\cdot 2^2\cdot 3^n\cdot 3}{7^n\cdot(1+7)} =$$

$$= \frac{4\cdot 3}{8}\cdot\sum_{n=0}^{\infty}\frac{2^n\cdot 3^n}{7^n} = \frac{3}{2}\cdot\sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n = \frac{3}{2}\cdot\frac{1}{1-\frac{6}{7}} = \underline{\underline{\frac{21}{2}}}$$

Zkouška: $(2)'' - 3\cdot 2' = 0$

(6.) $y'' - 3y' = 0$

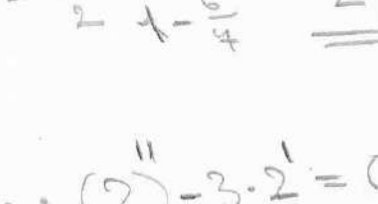
$\lambda^2 - 3\lambda = 0$

$\lambda(\lambda-3) = 0$

$\lambda = 0 \lor \lambda = 3$

F.S.: $1, e^{3x}$

obecné řešení: $y = c_1 + c_2\cdot e^{3x}$, $c_1, c_2 \in R$



$y(0)=2$: $2 = c_1 + c_2$

$y'(0)=0$: $0 = 3c_2$

$\Rightarrow c_2 = 0,\ c_1 = 2$

řešení $\boxed{y=2}$