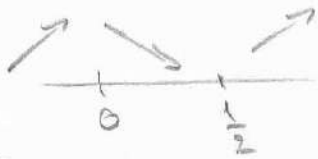
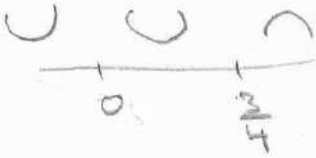
1) $f(x)=\frac{3x^2-4x+1}{x^2}=3-\frac{4}{x}+\frac{1}{x^2}$ $\qquad D_f=\mathbb{R}\setminus\{0\}$

$f'(x)=\frac{4}{x^2}-\frac{2}{x^3}=\frac{4x-2}{x^3}=0 \iff x=\frac{1}{2}$



$f''(x)=-\frac{8}{x^3}+\frac{6}{x^4}=\frac{-8x+6}{x^4}=0 \iff x=\frac{3}{4}$



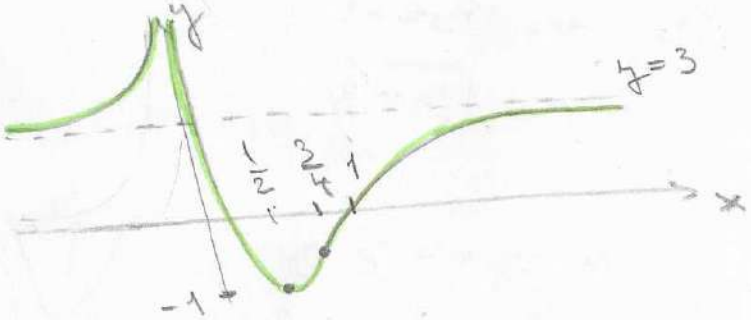
$f(\frac{1}{2})=3-8+4=-1$

$f(\frac{3}{4})=3-\frac{16}{3}+\frac{16}{9}=\frac{27-48+16}{9}=-\frac{5}{9}$

$f(x)=0 \iff 3x^2-4x+1=0$

$D=16-4\cdot3\cdot1=4$

$x_{1,2}=\frac{4\pm2}{6}=\Big<\begin{matrix}1\\ \frac{1}{3}\end{matrix}$



$\lim_{x\to0} f(x)=\frac{1}{0_+}=+\infty$

$\lim_{x\to\pm\infty} f(x)=3$

$H_f=\langle -1,\infty)$

2a) $\int \frac{1}{x\ln^3 x}\,dx=\left|\begin{matrix}\ln x=t\\ \frac{1}{x}dx=dt\end{matrix}\right|=\int\frac{1}{t^3}dt=\frac{t^{-2}}{-2}+c=-\frac{1}{\ln^2 x}+c$

2b) $\int x\sin(2x)\,dx=-\frac{1}{2}x\cos(2x)+\frac{1}{2}\int\cos(2x)\,dx=$

$\left|\begin{matrix}u=x & v'=\sin(2x)\\ u'=1 & v=-\frac{1}{2}\cos(2x)\end{matrix}\right.$ $\qquad -\frac{1}{2}x\cos(2x)+\frac{1}{4}\sin(2x$

2c) $\int_0^1(x-\sqrt{x})^2\,dx=\int_0^1(x^2-\underbrace{2x\sqrt{x}}_{x^{\frac{3}{2}}}+x)\,dx=\left[\frac{x^3}{3}-2\cdot\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+\frac{x^2}{2}\right]_0^1$

$=\frac{1}{3}-\frac{4}{5}+\frac{1}{2}=\frac{10-24+15}{30}=\frac{1}{30}$

2d) $\displaystyle\int \frac{x^3}{x^2-3x+2}\,dx = \frac{x^2}{2} + 3x + 8\ln|x-2| - \ln|x-1| + c$

$$x^3 : (x^2-3x+2) = x + 3 + \frac{7x-6}{x^2-3x+2}$$

$$\begin{array}{l} -(x^3-3x^2+2x) \\ \hline 3x^2-2x \\ -(3x^2-9x+6) \\ \hline 7x-6 \end{array}$$

$$\frac{7x-6}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$$

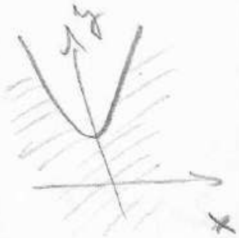
$$7x-6 = A(x-1) + B(x-2)$$

$\underline{x=1}$: $\quad 1 = 0A + (-1)B \implies \boxed{B=-1}$

$\underline{x=2}$: $\quad 8 = 1A + 0B \implies \boxed{A=8}$

3) $f(x,y) = \sqrt{x^2-y+4}$ $\qquad x^2-y+4 \geq 0$

$\qquad y \leq x^2+4$



$f(0,0) = \sqrt{4} = 2$

$$\frac{\partial f}{\partial x} = \frac{2x}{2\sqrt{x^2-y+4}}\Bigg|_{[0,0]} = 0$$

$$\frac{\partial f}{\partial y} = + \frac{(-1)}{2\sqrt{x^2-y+4}}\Bigg|_{[0,0]} = -\frac{1}{4}$$

tečná rovina: $\quad z-2 = -\frac{1}{4}y$

4) $f(x,y) = xy^2 - 4y - x$

$$\frac{\partial f}{\partial x} = y^2 - 1 = 0 \iff y = \pm 1$$

$$\frac{\partial f}{\partial y} = 2xy - 4 = 0$$

I. $y = 1$: $\quad 2x - 4 = 0$
$\qquad x = 2$

stac. body: $[2,1]$
$[-2,-1]$

II. $y = -1$: $\quad -2x - 4 = 0$
$\qquad x = -2$

$$H(x,y) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} = \begin{pmatrix} 0 & 2y \\ 2y & 2x \end{pmatrix}$$

$$\left.\begin{aligned} H(2,1) &= \begin{pmatrix} 0 & 2 \\ 2 & 4 \end{pmatrix} \\ H(-2,-1) &= \begin{pmatrix} 0 & -2 \\ -2 & -4 \end{pmatrix} \end{aligned}\right\} \det H < 0 \Rightarrow \text{sedlové body}$$