1. $f(x) = x + 2\sqrt{-x} + 3$ $\qquad D_f = (-\infty, 0\rangle$

$f'(x) = 1 - \frac{1}{\sqrt{-x}} = 0 \iff x = -1$

x	0	-1	-9
y	3	4	0

$\nearrow \quad \searrow$

$f'>0 \quad -1 \quad f'<0 \quad 0$ $\qquad$ lok. max. $[-1;4]$

$f''(x) = -\frac{1}{2}\cdot(-x)^{-\frac{3}{2}} < 0 \quad \forall x \in (-\infty, 0)$

$\Rightarrow$ f je konkávní na D_f $\qquad \cap$ $\quad f''<0 \quad 0$

$\underline{x=0}$: $f(0) = 0 + 2\cdot\sqrt{0} + 3 = 3$

$\underline{y=0}$: $x + 2\sqrt{-x} + 3 = 0$

$2\sqrt{-x} = -x - 3 \quad |^2$

$4\cdot(-x) = x^2 + 6x + 9$

$x^2 + 10x + 9 = 0$

$(x+9)(x+1) = 0$

$\underline{x = -9} \lor \underline{x = -1}$

Zkouška:

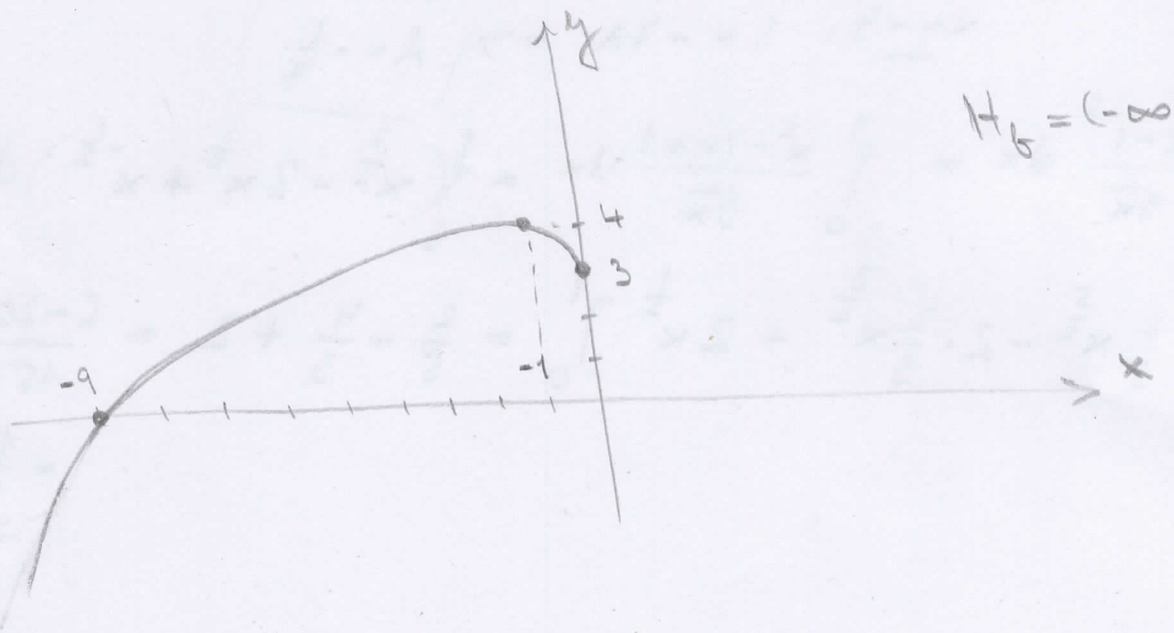
$x = -9$: $6 = 6$ ✓

$x = -1$: $2 = -2$ ×

$P_x = [-9; 0]$

$\lim\limits_{x\to-\infty}(x + 2\sqrt{-x} + 3) = \lim\limits_{x\to-\infty} x\cdot\left(1 + \frac{2\sqrt{-x}}{x} + \frac{3}{x}\right) = -\infty\cdot(1+0+0) = \underline{\underline{-\infty}}$

$H_f = (-\infty, 4\rangle$



(2a) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}\,dx = \int_0^1 \frac{dt}{\sqrt{t}} = \left[2t^{\frac{1}{2}}\right]_0^1 = \underline{\underline{2}}$

$\left|\begin{array}{l} \sin x = t \\ \cos x\,dx = dt \end{array}\right|$

(2b) $\int_0^{\infty} x e^{-x}\,dx \overset{P.P.}{=} \left[-xe^{-x}\right]_0^{\infty} + \int_0^{\infty} e^{-x}\,dx =$

$\left|\begin{array}{ll} u = x & v' = e^{-x} \\ u' = 1 & v = -e^{-x} \end{array}\right|$

$= \underbrace{\lim_{x\to\infty}\left(-\frac{x}{e^x}\right)}_{\overset{L.P.}{=} \lim\left(-\frac{1}{e^x}\right) = 0} - 0 + \left[-e^{-x}\right]_0^{\infty} = 0 + \underbrace{\lim_{x\to\infty}\left(-e^{-x}\right)}_{0} - \underbrace{\left(-e^0\right)}_{-1}$

$= 0 + 0 - (-1) = \underline{\underline{1}}$

(2c) $\int \frac{1}{x^2+x-12}\,dx = \frac{1}{7}\left(-\int \frac{1}{x+4}\,dx + \int \frac{1}{x-3}\,dx\right) =$

$= \frac{1}{7}\left(-\ln|x+4| + \ln|x-3|\right) + C = \underline{\frac{1}{7}\ln\frac{|x-3|}{|x+4|} + C}$

$\frac{1}{x^2+x-12} = \frac{1}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}$

$1 = A(x-3) + B(x+4)$

$\underline{x=3}: \quad 1 = 7B \Rightarrow \boxed{B = \frac{1}{7}}$

$x = \underline{-4}: \quad 1 = -7A \Rightarrow \boxed{A = -\frac{1}{7}}$

(2d) $\int_0^1 \frac{(x-1)^2}{\sqrt{x}}\,dx = \int_0^1 \frac{x^2-2x+1}{\sqrt{x}}\,dx = \int_0^1 \left(x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx$

$= \left[\frac{2}{5}x^{\frac{5}{2}} - 2\cdot\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}\right]_0^1 = \frac{2}{5} - \frac{4}{3} + 2 = \frac{6-20+30}{15}$

$= \underline{\underline{\frac{16}{15}}}$

(3.) $f(x,y) = x^3 \cos(xy)$

$f(2,\pi) = 8\cos(2\pi) = \underline{\underline{8}} = z_0$

$\frac{\partial f}{\partial x} = 3x^2\cos(xy) - x^3 y \sin(xy) \Big|_{[2,\pi]} = \underline{\underline{12}}$

$\frac{\partial f}{\partial y} = -x^4 \sin(xy) \Big|_{[2,\pi]} = \underline{\underline{0}}$

tečná rovina: $z - 8 = 12\cdot(x-2) + 0\cdot(y-\pi)$

$z = 12x - 16$

(4.) $f(x,y) = y^3 + x^2 + 15y^2 - 6xy - 4x + 12y$

$\frac{\partial f}{\partial x} = 2x - 6y - 4 = 0 \quad |:2$ $\qquad \boxed{x = 3y + 2}$

$\frac{\partial f}{\partial y} = 3y^2 + 30y - 6x + 12 = 0 \quad |:3$ $\qquad$ dosazení

$y^2 + 10y - 2(3y+2) + 4 = 0$

$y^2 + 4y = 0$

$y(y+4) = 0$

$\underline{y=0} \quad \vee \quad \underline{y=-4}$

$\underline{x=2} \qquad\quad \underline{x=-10}$

$\frac{\partial^2 f}{\partial x^2} = 2$

$\frac{\partial^2 f}{\partial y^2} = 6y + 30$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = -6$

HESSOVA MATICE

$H(x,y) = \begin{pmatrix} 2 & -6 \\ -6 & 6y+30 \end{pmatrix}$

$H(2,0) = \begin{pmatrix} 2 & -6 \\ -6 & 30 \end{pmatrix}$ $\qquad \begin{vmatrix} 2 & -6 \\ -6 & 30 \end{vmatrix} = 60 - 36 = 24 > 0$

$\wedge \ \frac{\partial^2 f}{\partial x^2} > 0$

$\Rightarrow [2;0]$ je lok. min.

$H(-10,-4) = \begin{pmatrix} 2 & -6 \\ -6 & 6 \end{pmatrix}$ $\qquad \begin{vmatrix} 2 & -6 \\ -6 & 6 \end{vmatrix} = 12 - 36 < 0$

$\Rightarrow [-10,-4]$ je sedlo