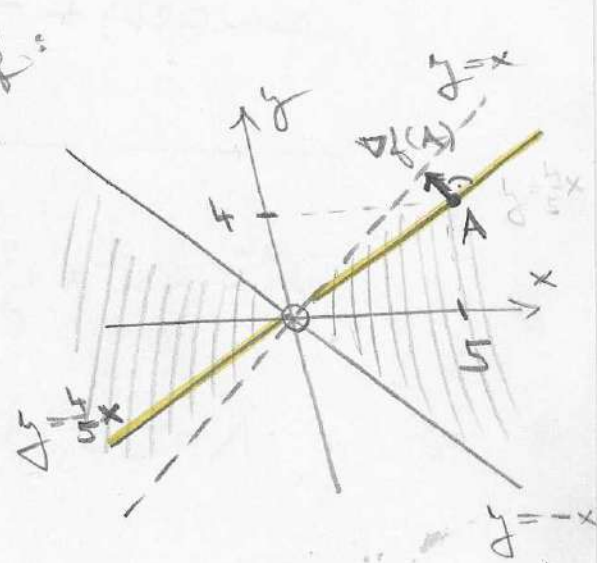


10. minitest
 Matematika M2 (b), LS 2025/26
 27. 4. 2026



Je dána funkce

$$f(x, y) = \sqrt{\frac{x+y}{x-y}}$$

- Určete a zakreslete její definiční obor.
- Vypočtěte gradient v bodě $A = [5, 4]$.
- Určete a popište vrstevnici ke grafu funkce f v bodě A . Zakreslete do jednoho obrázku definiční obor, gradient i vrstevnici.

$$a) \frac{x+y}{x-y} \geq 0 \iff ((x+y \geq 0) \wedge (x-y > 0)) \vee ((x+y \leq 0) \wedge (x-y < 0))$$

$$\iff ((y \geq -x) \wedge (y < x)) \vee ((y \leq -x) \wedge (y > x))$$

$$b) \frac{\partial f}{\partial x}(5, 4) = \frac{1}{2} \cdot \left(\frac{x+y}{x-y}\right)^{-\frac{1}{2}} \cdot \frac{x-y - (x+y)}{(x-y)^2} \Big|_{[5, 4]} = \frac{1}{2} \cdot 9^{-\frac{1}{2}} \cdot \frac{-8}{12} = -\frac{4}{3}$$

$$\frac{\partial f}{\partial y}(5, 4) = \frac{1}{2} \cdot \left(\frac{x+y}{x-y}\right)^{-\frac{1}{2}} \cdot \frac{x-y + (x+y)}{(x-y)^2} \Big|_{[5, 4]} = \frac{1}{2} \cdot 9^{-\frac{1}{2}} \cdot 10 = \frac{10}{6} = \frac{5}{3}$$

$$\nabla f(A) = \left(-\frac{4}{3}, \frac{5}{3}\right)^T$$

c) $f(5, 4) = \sqrt{9} = 3$ Vrstevnice: $f(x, y) = 3$

$$\sqrt{\frac{x+y}{x-y}} = 3 \quad |^2$$

$$\frac{x+y}{x-y} = 9 \quad | \cdot (x-y)$$

$$x+y = 9x - 9y$$

$$10y = 8x$$

římka 1

$$y = \frac{4}{5}x \quad | \cdot x \neq 0$$