

7. minitest
 Matematika M2 (b), LS 2025/26
 8. 4. 2026

Určete řešení diferenciální rovnice

$$y'' + y = x^2$$

s počátečními podmínkami $y(0) = 1$ a $y'(0) = 0$. Proveďte zkoušku.

$$1) \quad \underline{y'' + y = 0}$$

$$\lambda^2 + 1 = 0$$

$$\lambda_{1,2} = \pm i$$

F.S. $\cos x, \sin x$

$$y_H = c_1 \cos x + c_2 \sin x,$$

$$c_1, c_2 \in \mathbb{R}$$

$$2) \quad \underline{y_P = Ax^2 + Bx + C}$$

$$y'_P = 2Ax + B$$

$$y''_P = 2A$$

$$\underline{y''_P + y_P = 2A + \underline{Ax^2 + Bx + C}} \\ = \underline{x^2 + 0x + 0}$$

$$\boxed{A=1}$$

$$\boxed{B=0}$$

$$2A + C = 0 \Rightarrow \boxed{C=-2}$$

Obecné řešení: $y = y_H + y_P = c_1 \cos x + c_2 \sin x + x^2 - 2$

$$y(0) = c_1 \underbrace{\cos 0}_1 + c_2 \underbrace{\sin 0}_0 + 0^2 - 2 = c_1 - 2 = 1 \Rightarrow \boxed{c_1=3}$$

$$y' = -c_1 \sin x + c_2 \cos x + 2x$$

$$y'(0) = -c_1 \underbrace{\sin 0}_0 + c_2 \underbrace{\cos 0}_1 + 0 = c_2 = 0 \Rightarrow \boxed{c_2=0}$$

Řešení: $\boxed{y = 3 \cos x + x^2 - 2}$

Zkouška: $y' = -3 \sin x + 2x$
 $y'' = -3 \cos x + 2$

$$y(0) = 3 \cos 0 + 0^2 - 2 = 3 - 2 = 1 \quad \checkmark$$

$$y'(0) = -3 \sin 0 + 2 \cdot 0 = 0 \quad \checkmark$$

$$y'' + y = \cancel{-3 \cos x + 2} + \cancel{3 \cos x + x^2 - 2} \\ = x^2$$