1. $f(x,y) = \frac{4}{x} + xy + 2x - \ln y^2$ $\qquad D_f = (\mathbb{R} \setminus \{0\})^2$

$$\frac{\partial f}{\partial x} = -\frac{4}{x^2} + y + 2 = 0$$

dosazení

$$\frac{\partial f}{\partial y} = x - \frac{2}{y} = 0 \iff \boxed{y = \frac{2}{x}}$$

$$-\frac{4}{x^2} + \frac{2}{x} + 2 = 0 \quad \Big| \cdot x^2$$

$$-4 + 2x + 2x^2 = 0 \quad \Big| : 2$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2 \ \vee \ x = 1$$

$$y = -1 \qquad y = 2$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{8}{x^3}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{2}{y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 1$$

HESSOVA MATICE

$$H(x,y) = \begin{pmatrix} \frac{8}{x^3} & 1 \\ 1 & \frac{2}{y^2} \end{pmatrix}$$

$$H(-2,-1) = \begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix}$$

$\det H = -3 < 0$

$\Rightarrow [-2,-1]$ je sedlový bod

$f(-2,-1) = -4$

$$H(1,2) = \begin{pmatrix} 8 & 1 \\ 1 & \frac{1}{2} \end{pmatrix}$$

$\det H = 3 > 0 \ \wedge \ \frac{\partial^2 f}{\partial x^2} > 0$

$\Rightarrow [1,2]$ je lok. minimum

$f(1,2) = 8 - \ln 4$

(2.) $F(x,y) = \left(\ln(xy+y), \sqrt{4-4x^2-y^2}\right)$

I. $xy + y > 0$

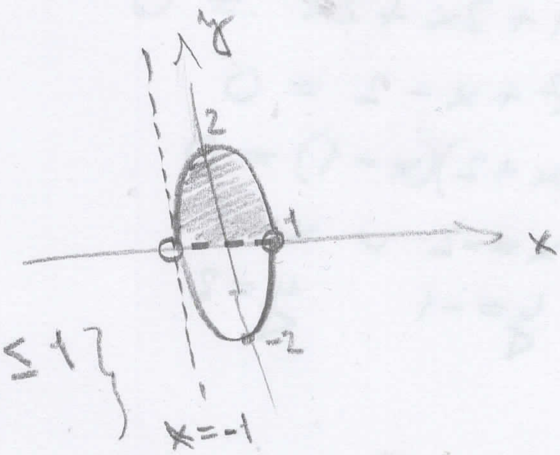
$y(x+1) > 0$

$((y>0) \wedge (x>-1))$
$\vee ((y<0) \wedge (x<-1))$

II. $4 - 4x^2 - y^2 \geq 0$

$4x^2 + y^2 \leq 4$

$x^2 + \frac{y^2}{4} \leq 1$



$D_F = \left\{(x,y) \in \mathbb{R}^2;\ x^2 + \frac{y^2}{4} \leq 1 \wedge y > 0 \right\}$

D_F není otevřená, neboť $[0,2] \in D_F$, ale $U_\varepsilon((0,2)) \not\subset D_F$
pro žádné $\varepsilon > 0$

je konvexní, neboť $\forall X, Y \in D_F : \overline{XY} \subset D_F$

$\ln(xy+y) = 0 \iff xy + y = 1$

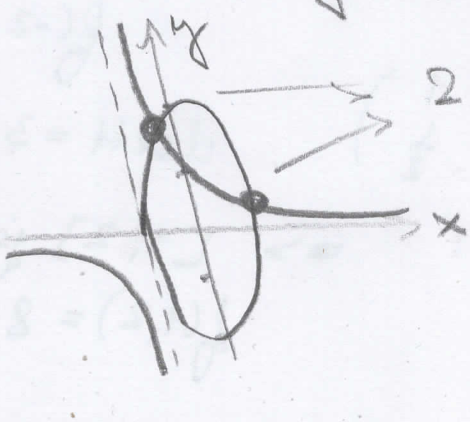
$\sqrt{4-4x^2-y^2} = 0$

$y(x+1) = 1$

$y = \frac{1}{x+1}$

$y = \frac{1}{x+1}$

$x^2 + \frac{y^2}{4} = 1$



2 řešení