1. $e^{x+y} - \frac{1}{y} + x + 1 = 0$

$$\boxed{f(-1) = 1}$$

$$e^{x+f(x)} - \frac{1}{f(x)} + x + 1 = 0 \quad \Big| \frac{d}{dx}$$

$$e^{x+f(x)} \cdot (1 + f'(x)) + \frac{1}{f^2(x)} \cdot f'(x) + 1 = 0$$

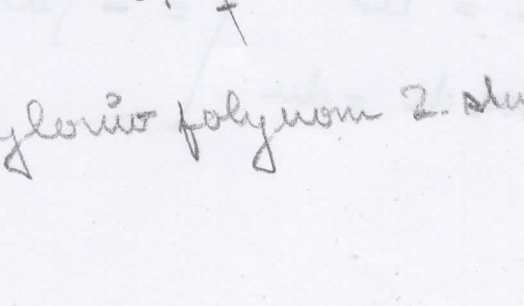
$$e^0 \cdot (1 + f'(-1)) + \frac{1}{1^2} \cdot f'(-1) + 1 = 0$$

$$2f'(-1) + 2 = 0 \implies \boxed{f'(-1) = -1}$$

$$\frac{d^2}{dx^2}: \quad e^{x+f(x)} \cdot (1 + f'(x))^2 + e^{x+f(x)} \cdot f''(x) + \frac{f''(x) \cdot f^2(x) - (f'(x))^2\, 2f(x)}{f^4(x)} = 0$$

$$e^0 \cdot (1-1)^2 + e^0 \cdot f''(-1) + \frac{f''(-1) - 2 \cdot (-1)^2 \cdot 1}{1} = 0$$

$$2f''(-1) - 2 = 0 \implies \boxed{f''(-1) = 1}$$



klesající, konvexní

Taylorův polynom 2. stupně: $\boxed{T^2_{f,-1}(x) = 1 + (-1) \cdot (x+1) + \frac{1}{2}(x+1)^2}$

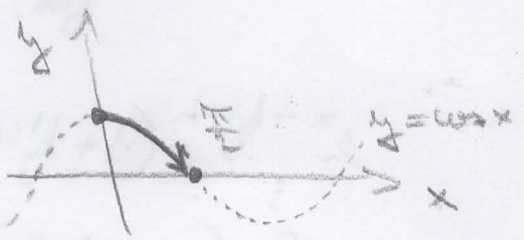
(2.)

$$\int_K (x-y)\,dx + y^2\,dy = \int_0^{\frac{\pi}{2}} (t-\cos t, \cos^2 t)\cdot(1, -\sin t)\,dt$$

$$\boxed{\int_K F = \int_\alpha^\beta (F \circ K(t)) \cdot K'(t)\,dt}$$

$$K(t) = (t, \cos t) \qquad , \quad t \in \langle 0, \tfrac{\pi}{2} \rangle$$

$$K'(t) = (1, -\sin t)$$



kladná orientace

$$= \int_0^{\frac{\pi}{2}} \left(t - \cos t - \cos^2 t \sin t \right) dt$$

$$\overset{(*)}{=} \left[\frac{t^2}{2} - \sin t + \frac{\cos^3 t}{3} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi^2}{8} - 1 - \frac{1}{3} = \underline{\underline{\frac{\pi^2}{8} - \frac{4}{3}}}$$

$$(*) \quad \int \cos^2 t \sin t\,dt = \left| \begin{array}{l} \cos t = u \\ -\sin t\,dt = du \end{array} \right| = -\int u^2\,du$$

$$= -\frac{u^3}{3} = -\frac{\cos^3 t}{3}$$