$$\int_0^1 \int_0^2 y x^2 e^{xy}\, dy\, dx \overset{P.P.}{=} \int_0^1 x^2 \cdot \left( \left[ y \cdot \frac{1}{x} e^{xy} \right]_{y=0}^{y=2} - \int_0^2 \frac{1}{x} e^{xy}\, dy \right)$$

$$\begin{vmatrix} u = y & v' = e^{xy} \\ u' = 1 & v = \frac{1}{x} e^{xy} \end{vmatrix}$$

PER-PARTES: $\boxed{\int uv' = uv - \int u'v}$

$$= \int_0^1 x^2 \cdot \left( \frac{2}{x} e^{2x} - \frac{1}{x} \cdot \left[ \frac{1}{x} e^{xy} \right]_0^2 \right) dx$$

$$= \int_0^1 \left( 2x e^{2x} - x^2 \cdot \frac{1}{x^2} \left( e^{2x} - 1 \right) \right) dx$$

$$= \int_0^1 2x e^{2x}\, dx - \int_0^1 \left( e^{2x} - 1 \right) dx$$

$$\overset{P.P.}{=} \left[ 2x \cdot \frac{1}{2} e^{2x} \right]_0^1 - \int_0^1 2 \cdot \frac{1}{2} e^{2x}\, dx - \left[ \frac{1}{2} e^{2x} - x \right]_0^1$$

$$\begin{vmatrix} u = 2x & v' = e^{2x} \\ u' = 2 & v = \frac{1}{2} e^{2x} \end{vmatrix}$$

$$= e^2 - \left[ \frac{1}{2} e^{2x} \right]_0^1 - \left( \left( \frac{1}{2} e^2 - 1 \right) - \frac{1}{2} \right)$$

$$= e^2 - \frac{1}{2} e^2 + \frac{1}{2} - \frac{1}{2} e^2 + 1 + \frac{1}{2} = 2$$