1. $f(x)=\log_{27}(6x-x^2) = (f_2\circ f_1)(x)$

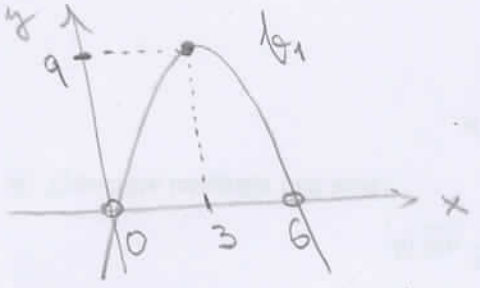
$f_1(x)=6x-x^2$

$f_2(x)=\log_{27}x$

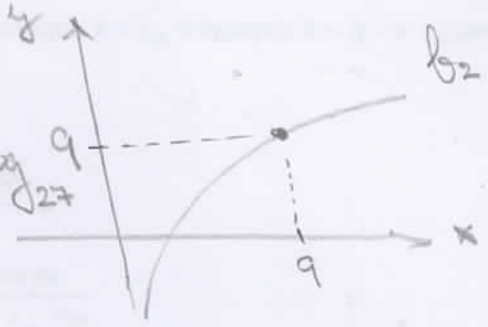
$6x-x^2>0$

$x(6-x)>0$

$x\in(0,6)$



$\frac{2}{3}=\log_{27}9$



$D_f=(0,6) \xrightarrow{f_1} (0,9\rangle \xrightarrow{f_2} (-\infty,\frac{2}{3}\rangle = H_f$

Funkce f není prostá, neboť např. $f(1)=f(5)=\log_{27}5$

tedy f^{-1} neexistuje

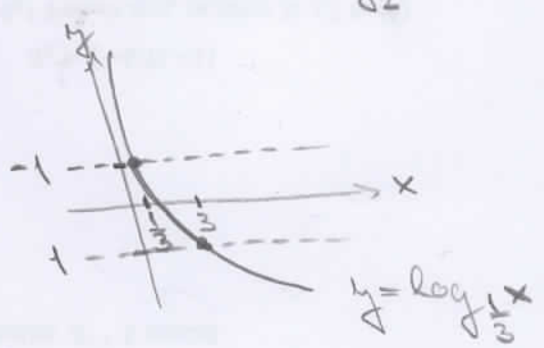
$g(x)=\arccos(\log_{\frac{1}{3}}x) = (g_2\circ g_1)(x)$

$g_1(x)=\log_{\frac{1}{3}}x$

$g_2(x)=\arccos x$

$x>0 \wedge -1\le \log_{\frac{1}{3}}x\le 1$

$\frac{1}{3}\le x\le 3$



$D_g=\langle\frac{1}{3},3\rangle \xrightarrow{g_1} \langle -1,1\rangle \xrightarrow{g_2} \langle 0,\pi\rangle = H_g$

$y=\arccos(\log_{\frac{1}{3}}x)$

$\cos y=\log_{\frac{1}{3}}x$

$\left(\frac{1}{3}\right)^{\cos y}=x \implies g^{-1}(x)=\left(\frac{1}{3}\right)^{\cos x}, x\in\langle 0,\pi\rangle$

2.

$$\lim_{x\to 0} \frac{\ln\left(\frac{1+\sin x}{1-\sin x}\right)}{\cos 3x \cdot \sin 2x} \overset{\text{L.P.}}{\underset{\text{"}\frac{0}{0}\text{"}}{=}} \lim_{x\to 0} \frac{\frac{1}{\frac{1+\sin x}{1-\sin x}} \cdot \left(\frac{1+\sin x}{1-\sin x}\right)'}{-3\sin 3x \cdot \sin 2x + 2\cos 3x \cdot \cos 2x}$$

$$= \lim_{x\to 0} \frac{\frac{1-\sin x}{1+\sin x} \cdot \frac{\cos x \cdot (1-\sin x) - (1+\sin x)\cdot(-\cos x)}{(1-\sin x)^2}}{-3\sin 3x \cdot \sin 2x + 2\cos 3x \cdot \cos 2x} =$$

$$= \frac{\frac{1-0}{1+0} \cdot \frac{1\cdot(1-0)-(1+0)\cdot(-1)}{(1-0)^2}}{-3\cdot 0\cdot 0 + 2\cdot 1\cdot 1} = \frac{1+1}{2} = \frac{2}{2} = \underline{\underline{1}}$$

3.

$$f(x) = \left(\frac{2}{1-x} + \frac{5}{2+x}\right) : \frac{3}{x^3+2x^2-x-2} =$$

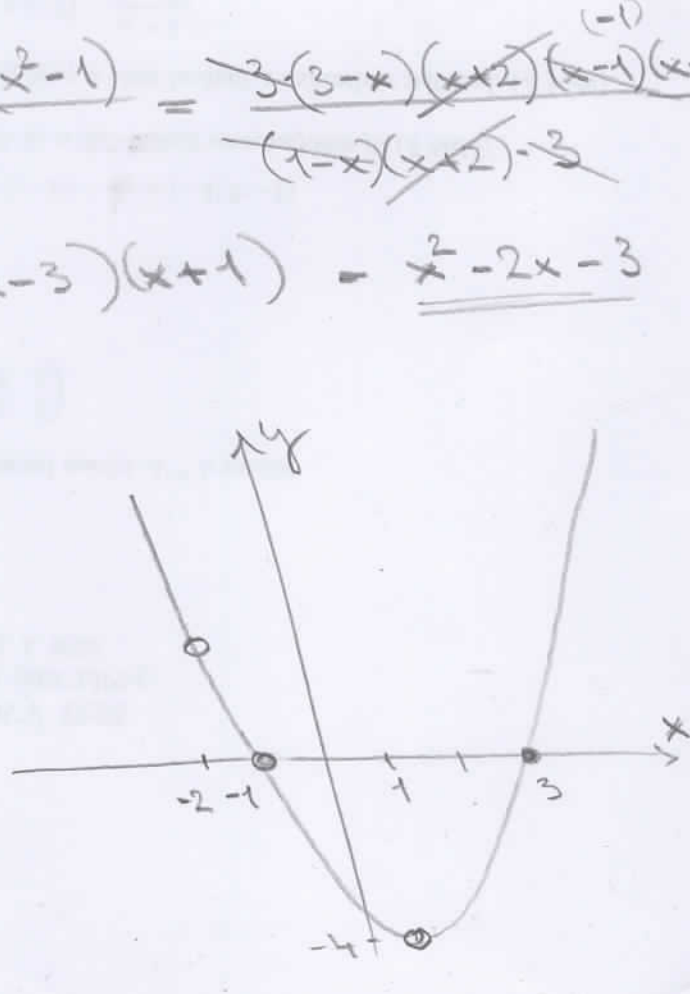
$$= \frac{2(2+x)+5(1-x)}{(1-x)(2+x)} \cdot \frac{x^2(x+2)-1\cdot(x+2)}{3} =$$

$$= \frac{9-3x}{(1-x)(x+2)} \cdot \frac{(x+2)(x^2-1)}{3} = \frac{\cancel{3}(3-x)\cancel{(x+2)}\overset{(-1)}{\cancel{(x-1)}}(x+1)}{\cancel{(1-x)}\cancel{(x+2)}\cdot\cancel{3}}$$

$$= (-1)(3-x)(x+1) = (x-3)(x+1) = \underline{\underline{x^2-2x-3}}$$

$D_f = \mathbb{R} - \{\pm 1; -2\}$

$H_f = (-4, \infty)$



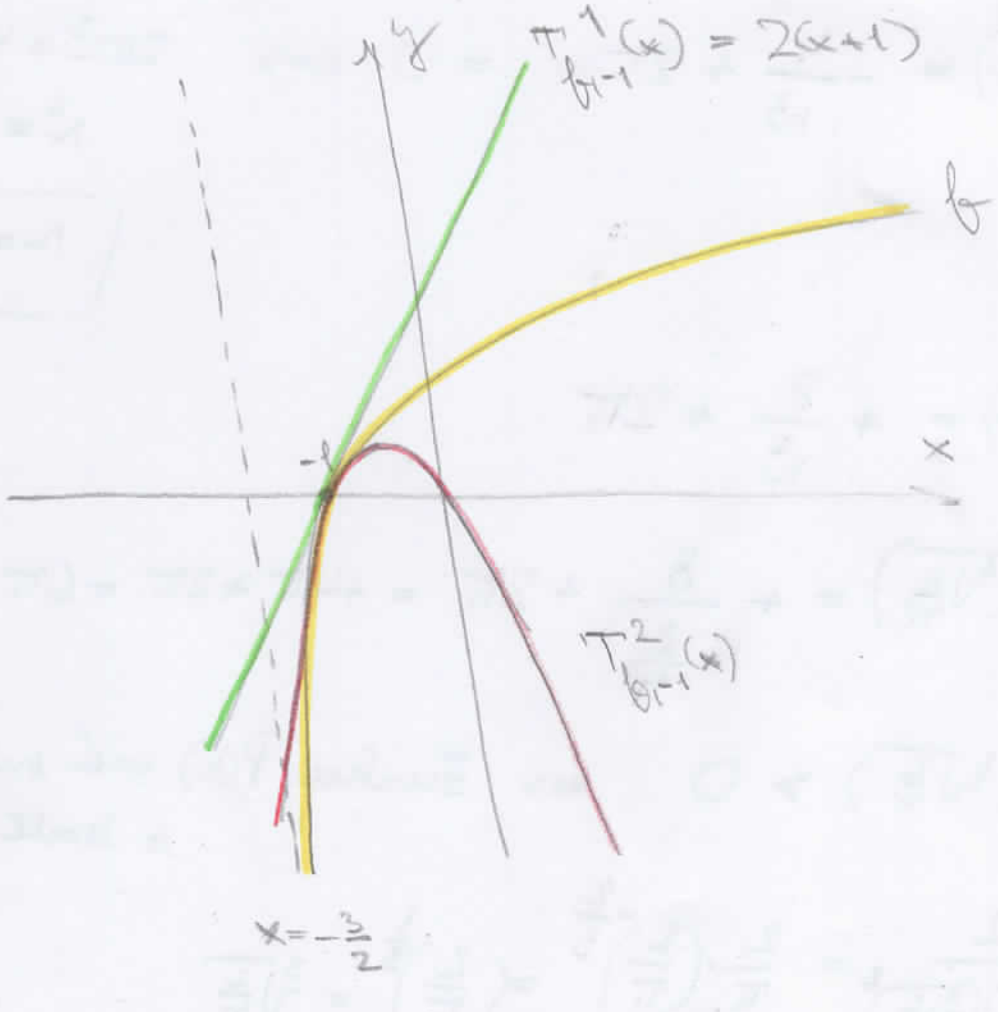
4. Taylorův polynom 2. stupně funkce f v bodě a má tvar: $T^2_{f,a}(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}\cdot(x-a)^2$

$$f(x) = \ln(3+2x) \Big|_{x=-1} = \ln 1 = 0 \qquad D_f = [-1;0]$$

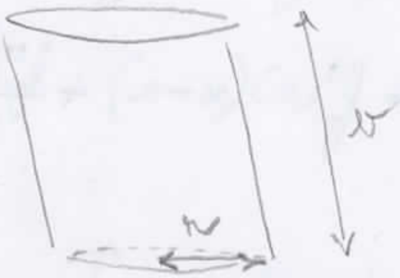
$$f'(x) = \frac{2}{3+2x} \Big|_{x=-1} = 2$$

$$f''(x) = -\frac{4}{(3+2x)^2} \Big|_{x=-1} = -4$$

$$T^2_{f,-1}(x) = 0 + 2(x+1) + \frac{(-4)}{2!}(x+1)^2 = \underline{\underline{2(x+1) - 2(x+1)^2}}$$



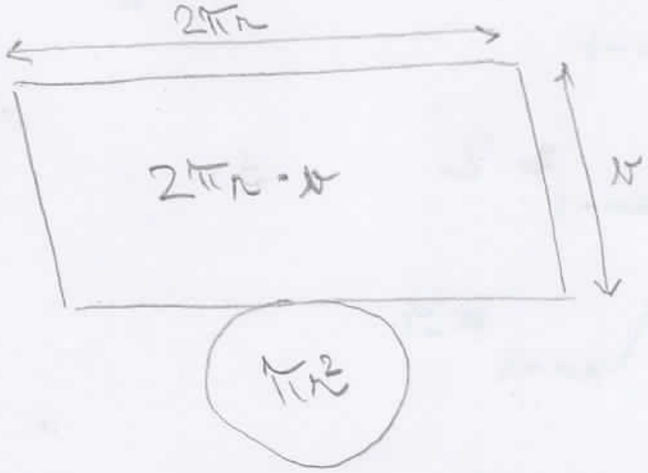
5.



Objem válce

$$V = \pi r^2 \cdot v = 2$$

$$\Longleftrightarrow \boxed{v = \frac{2}{\pi r^2}}$$



Povrch válce

$$P = 2\pi r \cdot v + \pi r^2$$

$$P(r) = 2\pi r \cdot \frac{2}{\pi r^2} + \pi r^2 = \frac{4}{r} + \pi r^2$$

$$P'(r) = -\frac{4}{r^2} + 2\pi r = 0 \Longleftrightarrow 2\pi r^3 = 4$$

$$r^3 = \frac{2}{\pi}$$

$$\boxed{r = \sqrt[3]{\frac{2}{\pi}}}$$

$$P''(r) = +\frac{8}{r^3} + 2\pi$$

$$P''\left(\sqrt[3]{\frac{2}{\pi}}\right) = +\frac{8}{\frac{2}{\pi}} + 2\pi = +4\pi + 2\pi = 6\pi$$

$P''\left(\sqrt[3]{\frac{2}{\pi}}\right) > 0 \implies$ Funkce $P(r)$ má minimum v bodě $\sqrt[3]{\frac{2}{\pi}}$

$$v = \frac{2}{\pi \cdot \left(\sqrt[3]{\frac{2}{\pi}}\right)^2} = \frac{2}{\pi} \cdot \left(\frac{2}{\pi}\right)^{-\frac{2}{3}} = \left(\frac{2}{\pi}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{2}{\pi}}$$

6) $f(x)=\dfrac{3x^2-4x+1}{x^2} = 3-\dfrac{4}{x}+\dfrac{1}{x^2}$ $\qquad D_f=\mathbb{R}\setminus\{0\}$

$\lim\limits_{x\to 0} f(x) = \dfrac{1}{0_+} = +\infty \Rightarrow$ svislá asymptota $x=0$

$\lim\limits_{x\to \pm\infty} f(x) = 3-0+0 = 3 \Rightarrow$ vodorovná asymptota v $\pm\infty$ je $y=3$

průsečíky s osami: $0 \notin D_f \Rightarrow$ neprotíná osu y

$f(x)=0 \iff 3x^2-4x+1=0$

$D = 16-4\cdot 3 = 4$

$x_{1,2} = \dfrac{4\pm 2}{6} = \begin{cases} 1 \\ \frac{1}{3} \end{cases}$ $\qquad P_{x_1}=[1,0] \quad P_{x_2}=[\frac{1}{3},0]$

$f'(x) = \left(3-\dfrac{4}{x}+\dfrac{1}{x^2}\right)' = \dfrac{4}{x^2}-\dfrac{2}{x^3} = \dfrac{4x-2}{x^3}$

$f'(x)=0 \iff 4x-2=0$

$x=\frac{1}{2}$

$\nearrow \quad \searrow \quad \nearrow$

$f'>0 \quad 0 \quad f'<0 \quad \frac{1}{2} \quad f'>0$

$f(\frac{1}{2}) = 3-\dfrac{4}{\frac{1}{2}}+\dfrac{1}{\frac{1}{4}} = 3-8+4 = -1$ $\qquad$ lok. min. $[\frac{1}{2}; -1]$

$f''(x) = -\dfrac{8}{x^3}+\dfrac{6}{x^4} = \dfrac{-8x+6}{x^4}$

$\cup \quad \cup \quad \cap$

$f''>0 \quad 0 \quad f''>0 \quad \frac{3}{4} \quad f''<0$

$f''(x)=0 \iff -8x+6=0$

$x = \frac{6}{8} = \frac{3}{4}$

inflexní bod $[\frac{3}{4}; -\frac{5}{9}]$

$f(\frac{3}{4}) = 3-\dfrac{4}{\frac{3}{4}}+\dfrac{1}{\frac{9}{16}} = 3-\dfrac{16}{3}+\dfrac{16}{9} = \dfrac{27-48+16}{9} = -\dfrac{5}{9}$

$H_f = \langle -1, \infty)$