

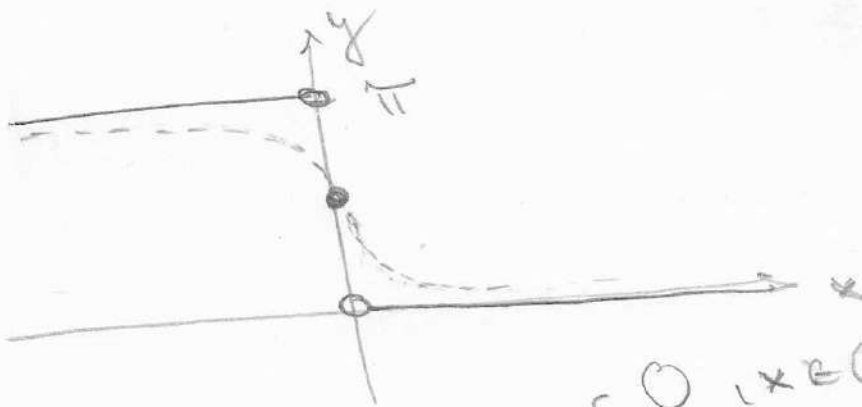
2. minitest RMF

Varianta A
9. 10. 2024

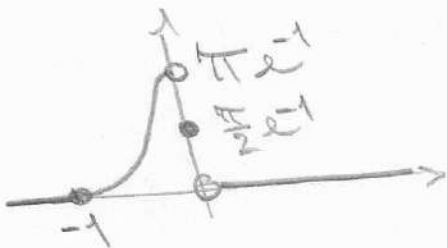
Rozhodněte a zdůvodněte, zda daná posloupnost funkcí konverguje v $\mathcal{D}(\mathbb{R})$.

$$f_n(x) = \begin{cases} \operatorname{arccotg}(nx) \cdot e^{-\frac{1}{1-x^2}} & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$

$$\lim_{n \rightarrow \infty} \operatorname{arccotg}(nx) = \begin{cases} \frac{\pi}{2} & x = 0 \\ 0 & x > 0 \\ \pi & x < 0 \end{cases}$$



$$\text{proto } \lim_{n \rightarrow \infty} f_n(x) = \begin{cases} 0 & x \in (-\infty, -1) \cup (0, \infty) \\ \frac{\pi}{2} \cdot e^{-1} & x = 0 \\ \pi \cdot e^{-\frac{1}{1-x^2}} & x \in (-1, 0) \end{cases}$$



$$\lim_{n \rightarrow \infty} f_n(x) = f(x) \notin C(\mathbb{R})$$

proto $f_n \not\rightarrow f$ a tedy $f_n \not\rightarrow f$ v $\mathcal{D}(\mathbb{R})$

2. minitest RMF

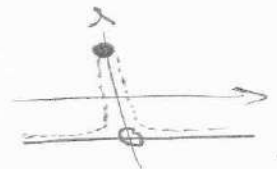
Varianta B
11. 10. 2024

Rozhodněte a zdůvodněte, zda daná posloupnost funkcí konverguje v $\mathcal{D}(\mathbb{R})$.

$$f_n(x) = \begin{cases} \cos\left(\frac{n\pi x^2}{1+n x^2}\right) \cdot e^{-\frac{1}{1-x^2}} & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$

$$\lim_{n \rightarrow \infty} \cos\left(\frac{n\pi x^2}{1+n x^2}\right) = \cos\left(\lim_{n \rightarrow \infty} \frac{\frac{\pi x^2}{1+x^2}}{\frac{1}{n} + x^2}\right) =$$

$$= \cos\left(\frac{\pi x^2}{0+x^2}\right) = \begin{cases} \cos \pi = -1, & x \neq 0 \\ 1, & x = 0 \end{cases}$$



proto $\lim_{n \rightarrow \infty} f_n(x) = \begin{cases} e^{-1}, & x = 0 \\ -e^{-\frac{1}{1-x^2}}, & |x| < 1, x \neq 0 \\ 0, & |x| \geq 1 \end{cases}$

$$\lim_{n \rightarrow \infty} f_n(x) = f(x) \notin \mathcal{C}(\mathbb{R})$$

tedy $f_n \not\rightarrow f$

a proto $f_n \not\rightarrow f$ v $\mathcal{D}(\mathbb{R})$

