1.

$$\begin{vmatrix} -2 & 0 & a \\ a & 3 & -8 \\ 0 & 1 & a \end{vmatrix} = -6a + a^2 - (0 + 16 + 0)$$

$$= a^2 - 6a - 16 = (a+2)(a-8)$$

$$\det A = 0 \iff \underline{a=-2} \lor \underline{a=8}$$

REGULÁRNÍ $\forall a \in \mathbb{R} \setminus \{-2; 8\}$

SINGULÁRNÍ pro $a=-2 \lor a=8$

2.

$$A = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot \operatorname{adj} A = \frac{1}{7\cdot 3 - 4\cdot 5} \cdot \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix}$$

3.

$$\begin{pmatrix} 3 & 8 \\ 2 & 3 \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 8 \\ 2 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 16$$

$$= (3-\lambda-4)(3-\lambda+4)$$

$$= (-1-\lambda)(7-\lambda) = 0$$

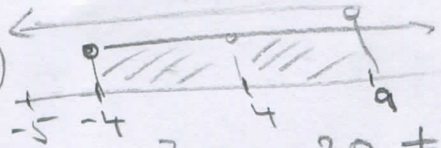
$$\iff \underline{\lambda = 7} \lor \underline{\lambda = -1}$$

$$M_7 = \operatorname{Ker}\begin{pmatrix} -4 & 8 \\ 2 & -4 \end{pmatrix} = \left\langle \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle$$

$$M_{-1} = \operatorname{Ker}\begin{pmatrix} 4 & 8 \\ 2 & 4 \end{pmatrix} = \left\langle \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\rangle$$

4.

$$f(x) = \frac{\sqrt{3x+12}}{x^2+x-20} + \log(18-2x)$$



• $18 - 2x > 0$
$18 > 2x \quad |:2$
$x < 9$

• $3x + 12 \geq 0$
$x \geq -4$

• $x^2 + x - 20 \neq 0$
$(x+5)(x-4) \neq 0$
$\underline{x \neq 4} \land \underline{x \neq -5}$

$$\boxed{D = \langle -4, 4) \cup (4, 9)}$$

5.

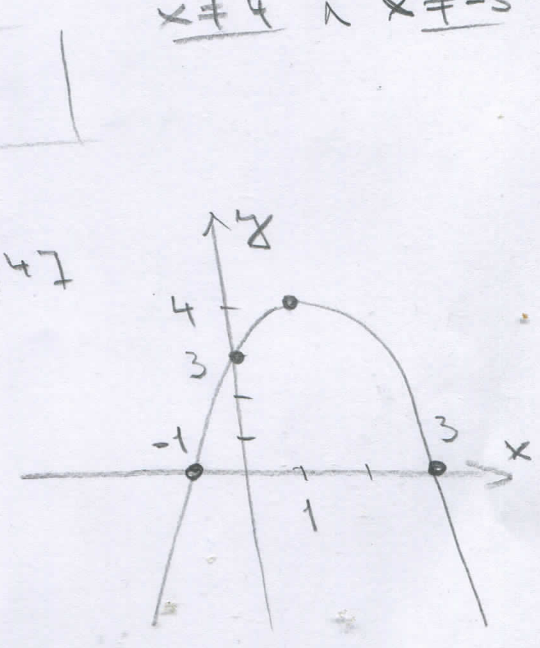
$$f(x) = -x^2 + 2x + 3$$

$$= -(x^2 - 2x - 3)$$

$$= -(x-3)(x+1)$$

$V = [1; 4]$

x	3	-1	0	1
$f(x)$	0	0	3	4



6. $f(x) = 2\arcsin\left(\frac{x}{4} - 3\right) + \frac{\pi}{2}$

$$-1 \le \frac{x}{4} - 3 \le 1 \quad | +3$$

$$2 \le \frac{x}{4} \le 4 \quad | \cdot 4$$

$$8 \le x \le 16$$

$$\boxed{D = \langle 8, 16 \rangle}$$

$$H_{\arcsin x} = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$$

$$H_{2\arcsin x} = \langle -\pi, \pi \rangle$$

$$\boxed{H = \langle -\frac{\pi}{2}, \frac{3}{2}\pi \rangle}$$

7. $$\lim_{x \to 3} \frac{18x - 2x^3}{x^2 + 2x - 15} = \lim_{x \to 3} \frac{2x \overset{(-1)}{\cancel{(3-x)}}(3+x)}{(x+5)\cancel{(x-3)}_1}$$

$$= \frac{{}^3\cancel{6} \cdot (-1) \cdot \cancel{6}^3}{\cancel{8}_{\cancel{4}_2}} = -\frac{9}{2}$$

$$\lim_{x \to \infty} \frac{5 - 18x^2}{6x^2 + x} = \lim_{x \to \infty} \frac{\cancel{x^2} \cdot \left(\frac{5}{x^2} - 18\right)}{\cancel{x^2} \cdot \left(6 + \frac{1}{x}\right)} = \frac{0 - 18}{6 + 0} = -3$$

$$\lim_{x \to \frac{\pi}{2}^+} e^{\frac{1}{\cos x}} = e^{\frac{1}{0^-}} = e^{-\infty} = 0$$