(3b) $f(x)=\dfrac{3x-2}{2x^2}$, $D_f=\mathbb{R}\setminus\{0\}$ $\qquad \lim\limits_{x\to 0_+} f(x)=\lim\limits_{x\to 0_-} f(x)=-\infty$

$\lim\limits_{x\to\infty} f(x)=0$, $\lim\limits_{x\to-\infty} f(x)=0$

průsečíky s osami: $[\frac{2}{3},0]$

$x=\frac{2}{3} \iff y=0$

1. derivace: $f'(x)=\dfrac{3\cdot 2x^2-(3x-2)\cdot 4x}{4x^4}=\dfrac{6x^2-12x^2+8x}{4x^4}=\dfrac{8x-6x^2}{4x^4}$

$f'(x)=0 \iff 8x-6x^2=0 \quad |:2$

$4x-3x^2=0$

$x(4-3x)=0 \iff \underline{x=0} \vee \underline{x=\frac{4}{3}}$

$\searrow \quad \nearrow \quad \searrow$

$f'<0 \quad 0 \quad f'>0 \quad \frac{4}{3} \quad f'<0$

$\forall x\in(-\infty,0)\cup(\frac{4}{3},\infty)$: $f'(x)<0$

$\Rightarrow$ funkce je klesající

x	$-$	$+$	$+$
$4-3x$	$+$	$+$	$-$
	$\ominus$	$\oplus$	$\ominus$

$-\infty \quad 0 \quad \frac{4}{3} \quad \infty$

$\forall x\in(0,\frac{4}{3})$: $f'(x)>0 \Rightarrow$ funkce je rostoucí

lokální maximum je v $x=\frac{4}{3}$, $f(x)=\dfrac{3\cdot\frac{4}{3}-2}{2\cdot\frac{16}{9}}=\dfrac{2}{2\cdot\frac{16}{9}}=\dfrac{9}{16}$

tj. $[\frac{4}{3};\frac{9}{16}]$ je lok. max.

2. derivace: $f''(x)=\dfrac{(8-12x)4x^4-(8x-6x^2)\cdot 16x^3}{16x^8}=\dfrac{(8-12x)x-(8x-6x^2)\cdot 4}{4x^5}=$

$=\dfrac{8x-12x^2-32x+24x^2}{4x^5}=\dfrac{-24x+12x^2}{4x^5}=\dfrac{-6+3x}{x^4}$

$f''(x)=0 \iff -6+3x=0$

$\underline{x=2}$ $\qquad f(2)=\dfrac{3\cdot 2-2}{8}=\underline{\dfrac{1}{2}}$

$\Rightarrow$ inflexní bod $[2;\frac{1}{2}]$

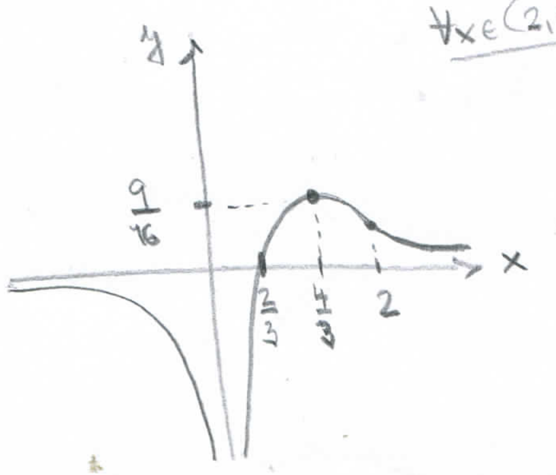
$\cap \quad \cap \quad \cup$

$f''<0 \quad 0 \quad f''<0 \quad 2 \quad f''>0$

$\forall x\in(-\infty,0)\cup(0,2)$: $f''(x)<0 \Rightarrow$ funkce je KONKÁVNÍ

$\forall x\in(2,\infty)$: $f''(x)>0 \Rightarrow$ funkce je KONVEXNÍ

Graf:



Obor hodnot $\boxed{H_f=(-\infty,\frac{9}{16}\rangle}$

3c) $f(x)=\dfrac{x^2}{x-1}$ $D_f=\mathbb{R}\setminus\{1\}$

$\lim\limits_{x\to 1_+} f(x)=\dfrac{1}{0_+}=+\infty$ $\lim\limits_{x\to\infty}\dfrac{x^2}{x-1}=\infty$ → L'HOSPITALOVO PRAVIDLO $\left(=\lim\limits_{x\to\infty}\dfrac{2x}{1}\right)$ "$\dfrac{\infty}{\infty}$"

$\lim\limits_{x\to 1_-} f(x)=\dfrac{1}{0_-}=-\infty$ $\lim\limits_{x\to -\infty}\dfrac{x^2}{x-1}=-\infty$

1. derivace: $f'(x)=\dfrac{2x(x-1)-x^2}{(x-1)^2}=\dfrac{2x^2-2x-x^2}{(x-1)^2}=\dfrac{x^2-2x}{(x-1)^2}$

$f'(x)=0 \iff x^2-2x=0$
$x(x-2)=0$
$\underline{x=0} \vee \underline{x=2}$ stacionární body

$\nearrow$ $\searrow$ $\nearrow$
$f'>0$ | 0 | $f'<0$ | 2 | $f'>0$

$\forall x\in(-\infty,0)\cup(2,\infty)$: $f'(x)>0 \Rightarrow$ funkce roste
$\forall x\in(0,2)$: $f'(x)<0 \Rightarrow$ funkce klesá

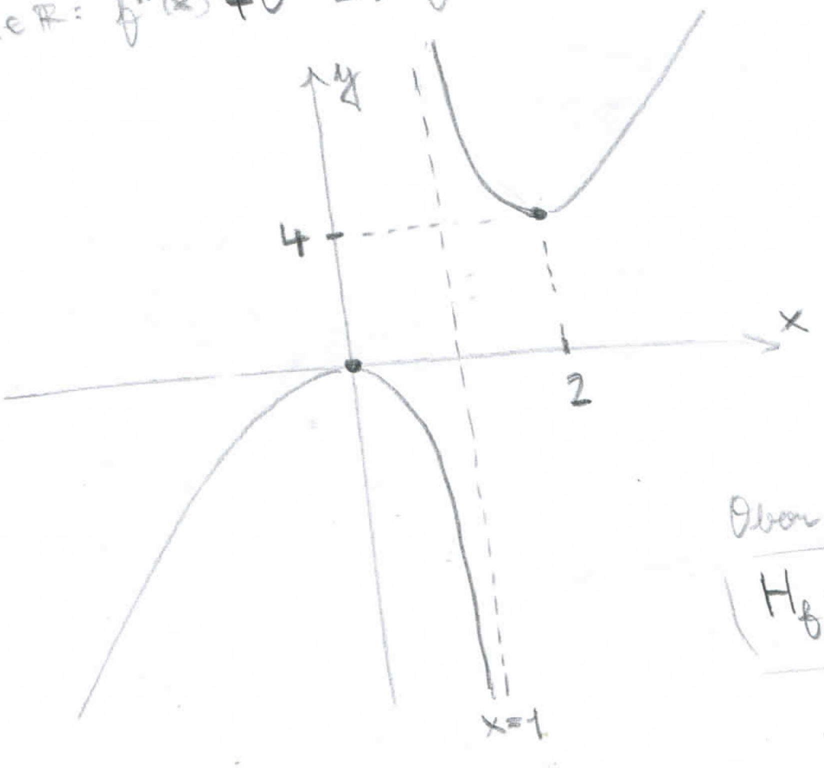
lok. maximum: $[0;0]$
lok. minimum: $[2;4]$

2. derivace: $f''(x)=\dfrac{(2x-2)(x-1)^2-(x^2-2x)\cdot 2(x-1)}{(x-1)^4}=\dfrac{(2x-2)(x-1)-2(x^2-2x)}{(x-1)^3}$

$=\dfrac{\cancel{2x^2}-\cancel{2x}-\cancel{2x}+2-\cancel{2x^2}+\cancel{4x}}{(x-1)^3}=\dfrac{2}{(x-1)^3}$

$\forall x\in(-\infty,1)$: $f''(x)<0 \Rightarrow$ funkce je KONKÁVNÍ
$\forall x\in(1,\infty)$: $f''(x)>0 \Rightarrow$ funkce je KONVEXNÍ
$\forall x\in\mathbb{R}$: $f''(x)\neq 0 \Rightarrow$ funkce nemá inflexní body

Graf:



Obor hodnot:

$H_f=(-\infty,0\rangle\cup\langle 2,\infty)$

(3d) $f(x) = \operatorname{arctg}\left(\frac{1}{x}\right)$ $\qquad D_f = \mathbb{R} \setminus \{0\}$

$$\lim_{x \to 0_+} f(x) = \arctan\left(\frac{1}{0_+}\right) = \frac{\pi}{2} \qquad \lim_{x \to \infty} f(x) = \arctan 0 = 0$$

$$\lim_{x \to 0_-} f(x) = \arctan\left(\frac{1}{0_-}\right) = -\frac{\pi}{2} \qquad \lim_{x \to -\infty} f(x) = \arctan 0 \doteq 0$$

$\forall x \in D_f : \arctan\left(\frac{1}{(-x)}\right) = -\arctan\left(\frac{1}{x}\right) \Rightarrow f$ je lichá funkce

1. DERIVACE:

$$f'(x) = \frac{1}{\left(\frac{1}{x}\right)^2 + 1} \cdot \left(-\frac{1}{x^2}\right) = -\frac{1}{1+x^2} \neq 0 \Rightarrow f \text{ nemá extrém}$$

$\forall x \in D_f : f'(x) < 0 \Rightarrow$ funkce je klesající na D_f

2. DERIVACE:

$$f''(x) = \frac{2x}{(1+x^2)^2} \qquad f''(x) = 0 \iff x = 0$$

$\forall x \in (0, \infty) : f''(x) > 0 \Rightarrow$ funkce je konvexní

$\forall x \in (-\infty, 0) : f''(x) < 0 \Rightarrow$ funkce je konkávní

Graf:

Obor hodnot: $\boxed{H_f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$